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發行宗旨

一、本刊為一實務性的數學教育刊物，出版目的如下：
1. 積極發揚台灣數學教育學會之成立宗旨：研究、發展、推廣數學教育，使台灣學生快樂學好數學。
2. 提升數學教師教學品質、數學教育研究品質及促進數學教學策略與方法之交流。
3. 探討數學教育的學術理論與實務現況，以促進理論與實務之結合，進一步提升數學教學之內涵。
4. 提供數學教育課程、教材與教法等實務經驗，包括數學遊戲、DIY 教具之分享，以供未來之教學與研究參考之用。
5. 針對多數學生特定迷思概念之教學引導，如學生易有的錯誤型態及如何釐清觀念等。
6. 介紹國內外數學教育現況。

二、本刊內容以充實高中、國中與小學數學教學、課程與教材為主，以提供所有關心數學教育人士之教學資源與參考依據。

三、本期刊以季刊方式（3 個月一期，一年共 4 期）發行，分別於每一年的 3、6、9、12 月發行。

四、本期刊採電子與紙本方式同時發行。

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EDITORIAL

It’s a beginning for the better future

This is a special issue, including five articles, that presented at the conference “The Perspective of Mathematics Curriculum Development and Teaching Innovation” organized by Der-Ching Yang, National Chiayi University. Two of them are invited papers from the Robert E. Reys, Curators’ professor of University of Missouri-Columbia, and Barbara J. Reys, Distinguished professor of University of Missouri-Columbia. Dr. Robert Reys introduced “Some Challenges Facing Mathematics Education in the United States of America.” It let us have more chance to know and compare the challenges we faced. In the article “What role do textbooks play in U.S. middle school mathematics classrooms?” Dr. Barbara Reys and her doctoral students reported that how textbooks impact U.S. students’ opportunity to learn mathematics in important ways. It leads us to understand how textbooks impact the students’ learning.

Dr. Shu-Ling Chang and Dr. Fou-Lai Lin investigated an elementary school teacher's strategies for advancing children’s mathematical thinking. It supports us a good example how a teacher help children advance their mathematical thinking when implemented the activities in the textbooks. Dr. Jung-Chih Chen examined the learning expectations related to Grade 1-8 geometry in some Asian countries and U.S. States. Finally, Dr. Ching-Shu Chen reported a detailed overview of the US-Russian and measure up curriculum. At the same time, it also examined how it can be implemented to help students build a solid foundation in elementary mathematics.

These articles summarize important results from different perspectives. We sincerely appreciate to all of the authors for your contributions to TJMT. Due to your help, TJMT will have better future.

This is the second year of TJMT to publish papers for teachers and mathematics educators in Taiwan. The TJMT needs your help to submit manuscripts and give comments. This is a special issue published by English version. Maybe it is inconvenient for some of the readers in here. However, it is also a good chance for you to read the different viewpoints from the international perspectives. I do believe it will help us to know more about the topics related to mathematics education and understand what happen on mathematics education around our world.

Finally, I do hope that it’s a good beginning for better future to TJMT due to your contributions.

Der-Ching Yang

Editor-in-Chief

Some Challenges Facing Mathematics Education in the United States of America

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Epilogue

This paper is based on a talk made at the National Chiayi University in Taiwan in January 2006. After presenting my list of challenges, I asked the audience to make a list of challenges mathematics educators are currently facing in Taiwan. As these discussions evolved, it became clear that many of the challenges I identified are also being faced in Taiwan. While solutions to these challenges are formidable, we know that one of the first steps in solving any problem is understanding and recognizing that a problem exists. In that spirit, I am sharing these challenges in a written form. Perhaps identifying these challenges will accelerate efforts to solve them.

Opening

Much is happening in mathematics education in the United States. Challenges abound, and the list is long. Here are five challenges I think need to be addressed:

- Implementing Standards
- Resolving the shortage of mathematics teachers
- Adapting to changing technology
- Making the best of mandated assessments
- Advancing knowledge with research

This is a short list. It could be longer. If other mathematics educators in the United States were asked to compile a list of challenges, their list would likely be different. However, I think the intersection of their lists would include many if not all of the above issues. Now here is some elaboration on these challenges.

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Implementing Standards

The United States has no national curriculum. In fact, education in the United States is viewed as a local responsibility. Historically each state and community has made their own decisions about what, when and how learning in schools should take place. There are 50 states and more than 25,000 school districts in our country. This approach has resulted in much duplication of effort as individual schools, school districts, and states have developed their own curriculum frameworks. This piecemeal approach has resulted in much variability of what and when specific mathematical topics are taught. For example, some states expect mastery of multiplication facts in third grade, while other state frameworks expect mastery in second grade, and still others in fourth grade (Reys, et. al., under review). This lack of consensus of when certain topics are taught has created havoc for textbook publishers and has resulted in much duplication of the mathematics content in textbooks across the grades.

In 1989 the National Council of Teachers of Mathematics released a document entitled Curriculum and Evaluation Standards for School Mathematics. This was a landmark document for our country. It was the first time that any professional organization had offered national recommendations related to curriculum. It identified content and process strands along with a vision for mathematics teaching and learning. This document was followed by Professional Standards for Teaching Mathematics (1991). These Standards stated that mathematics is for everyone, not just a select few. They also asserted that mathematics learning should be characterized by sense making. These Standards have resulted in some significantly different mathematics curricula becoming available. They also advocated approaches to mathematics teaching that were unfamiliar to many teachers.

The 1989 Standards were followed in 2000 with an updated version entitled Principles and Standards for School Mathematics. This document provided updates regarding mathematics content and processes that need to be addressed. These standards were identified:

Problem Solving

Communication

Reasoning

Connections

Representation

Number and Operation

Algebra

Geometry

Measurement

Data Analysis and Probability

The content of these Standards documents provided an ambitious vision for mathematics curriculum and teaching that is far from being realized in the United States. For more than a decade, widespread implementation of these Standards in classrooms across the United States has been a challenge, and there is no end in sight.

Resolving the shortage of mathematics teachers

For more than 20 years there has been a shortage of mathematics teachers in the United States. There are multiple reasons for the teacher shortage, and here are some of them:

- An increasing number of students in schools in the United States.
- More students are taking more courses in mathematics in secondary school.
- Fewer people are preparing to be mathematics teachers.
- Many people who might have become mathematics teachers have jobs in business, computer science, statistics, related technologies and industry.
- More mathematics teachers are retiring than are entering the profession.

This shortage has resulted in many mathematics classes being taught by teachers with limited mathematics background. For example the National Center for Education Statistics reports that nearly 70% of middle/secondary mathematics teachers in the United States lack a major in or certification in mathematics, and 95% of large urban school districts have an immediate need for mathematics teachers. This challenge has resulted in many large school districts recruiting mathematics teachers from other

English speaking countries, such as India. Recruiting teachers from other countries is only a bandaid approach to a challenge that requires a long term fix.

Adapting to changing technology

There is general agreement that appropriate use of technology should be an integral part of school mathematics programs. This includes a wide spectrum of tools for classroom use, including graphing calculators, spreadsheets, Geometer's Sketchpad, and Mathematicia. While support for technological tools is strong their availability and actual use in mathematics classrooms varies greatly due to teacher competence in using these tools as well as their availability.

Calculator use in elementary schools continues to be sporadic. The National Council of Teachers of Mathematics has a position statement on use of calculators and technology (available at www.nctm.org). A portion of the statement says:

> "when calculators are used effectively in the classroom, they can enhance students' understanding and use of numbers and operations. Teachers can capitalize on the appropriate use of this technology to expand students' mathematical understanding, not to replace it."

Striving for calculators to be "used effectively" remains a challenge. It should also be noted that while calculators are generally available, and are used in middle and secondary schools, calculator use in elementary school is much more varied. It is not unusual to find some elementary teachers using calculators in their classes, and other elementary teachers in the same school prohibiting the use of calculators. The same unpredictable use of calculator is seen in colleges and universities across the country. Some mathematics departments allow calculators to be used, others prohibit their use. Overall the investment in professional development of calculator use has been insufficient in the United States. Consequently, effective, consistent and widespread use of calculators in elementary school and colleges has not been achieved in the United States.

Making the best of mandated assessments

In 1995 the National Council of Teachers of Mathematics published Assessment Standards for School Mathematics. That document discussed different ways of using assessments to promote and guide better learning of mathematics. While participation in international assessments and achievement tests have long been a part of the culture in the United States, during the last decade there has been an growing number of mandated assessments. These mandated tests have been developed at the state level and have typically focused on reading and mathematics. The results from these tests are widely published, and often cited when teachers and schools are evaluated.

In 2001, a new federal law entitled No Child Left Behind was passed. This law applied additional pressure on states and all schools within each state to have annual assessments that would be used to document annual growth of ALL students--including different sub-groups, such as gender and ethnic groups. Growth must be continuous over time (i.e., from one year to the next) otherwise schools are subject to potentially severe sanctions, including the loss of federal funds. The alignment of these state tests with individual school curriculum varies greatly. Consequently these mandated tests have placed great pressure on schools and classroom teachers for students to do well on tests over which the teachers have no control. Furthermore, the test scores do not generally impact individual student grades, so there is little motivation for students to do well on the tests. How to cope with this heavy emphasis on testing that is beyond teacher control is a major challenge.

Advancing knowledge with research

There has been a growing interest in research to guide educational decisions. Schools are looking for research evidence to guide practical decisions, such as

- What mathematics curriculum to use?
- Is the way mathematics is taught important?
- How should mathematics classes be organized to promote student learning?
- Do boys learn different than girls?
- How much homework is needed to anchor certain skills?

The list of practical questions for which mathematics teachers are seeking answers is virtually unlimited. And researchers in mathematics education are interesting in studying these issues in a systematic fashion. This would appear to be the ideal climate for initiating much research in mathematics education. So what is the problem?

First of all research in mathematics education is complex. There are many factors that influence outcomes, so even answers to what seem like simple and direct questions are difficult to produce (Berliner, 2002). Furthermore, there are very limited funds available to conduct research studies in education, and the United States Department of Education has set a “gold standard” for educational studies that are federally funded. Specifically the gold standard draws on the medical model and requires randomization to be an integral part of the research design. The randomization requirement makes it virtually impossible to get schools to cooperate in such research. In addition, research studies require additional time to collect data from students, via pre and post measures. This type of data collection imposes on actual instructional time. Teachers and principals are reluctant to sacrifice instructional time to research efforts. Thus designing and conducting school based research studies is a challenge, yet one that needs to be resolved if the frontier of knowledge in mathematics education is to be advanced.

Conclusion

During my talk, it became clear that at least one of the challenges in the United States are not a challenge in Taiwan. For example, while we have a shortage of certified mathematics teachers, Twaiwn has an abundance of teachers. For those of you that are excellent English speakers, there is a great opportunity for you to teach mathematics in the United States!

The challenges I highlighted here, related to Standards, Teacher shortages, Technology, Testing, and Research are not going to go away any time soon for us in the United States. They are complex. They involve many people. They require professional development. More money is needed to help address them. Having said that, the bright side is that these challenges are a reminder that our mathematics education community has much to do to resolve them. Our careers are built on the journey as we work toward solutions provided by the opportunities presented by these challenges.

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What role do textbooks play in U.S. middle school mathematics classrooms?

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What role do textbooks play in U.S. middle school mathematics classrooms?

U.S. school districts spend about 600 million dollars annually on mathematics textbooks (Education Market Research, 2005). These textbooks are typically not consumable. That is, students are loaned the books, they cannot write in them, and they must turn them back into the school at the end of the year for use by other students in subsequent years. Textbook pages are generally printed on high quality paper using multiple colors for graphics and are bound in hard covers to extend the life of the materials as school districts generally use the adopted textbook series for six to eight years before purchasing new textbooks.

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U.S. mathematics textbooks have grown in size over the past 20 years with most current textbooks exceeding 750 pages in length for a given grade level. Textbooks generally include much more material than teachers or students can cover in a given school year. This is due primarily to the fact that the U.S. has no national curriculum. Rather, each state determines curriculum standards and there is considerable variation in content focus at particular grades across the many state-level standards (Reys, in press). Therefore, publishers include lessons on mathematics content in a given grade-level textbook that meet multiple, varying state standards.

Research has documented that U.S. teachers rely heavily on the district-adopted textbook to make decisions about what content to teach and when to teach it. In fact, nearly three-fourths of 8th grade teachers in the U.S. report using their textbook on a daily basis (Grouws and Smith, 2000), while two-thirds of middle grade mathematics teachers indicate they "cover" at least three-fourths of the textbook in a given year (Weiss, et al., 2001). Conversely, some teachers ignore their school-purchased textbook (Seeley, 2003) and create their own instructional materials based on their experiences and beliefs about what mathematics is important and how it should be taught. Even teachers who typically use their textbooks do so in very different ways (Chávez, 2003). Much of this variance can be attributed to the fact that U.S. teachers are provided considerable autonomy in making decisions about classroom practice. Therefore, students in the same school or district often experience a different mathematics curriculum, depending on decisions made by their teacher.

Although they are used in various ways and to different extents, for most students mathematics textbooks shape the activity of the classroom and influence opportunity to learn mathematics (Reys et al. 2003; Porter, 1989). This is due, in part, to the belief that the subject of mathematics is very hierarchical in nature and must be logically ordered and presented. At the elementary and middle school levels, many teachers feel less confident teaching mathematics than other subjects and are therefore more likely to lean on the textbook for advice and direction.

This article reports findings from a study that monitored use of district-adopted mathematics textbooks in selected U.S. middle schools over a two-year period. It summarizes the mathematical content focus of district-adopted textbooks, the material from textbooks that was used by teachers, and the material that was typically omitted.

About the Study

Eleven middle schools in six states were selected to participate in this study based on their choice of district-adopted textbook, the length of its use in the school district, and the willingness of administrators and teachers to participate in the research (see Table 1 for a complete list of textbooks used in the participating schools). An effort was made to identify school districts using the most popular (widely used) mathematics textbooks in the U.S., as identified by the 2000 Mathematics and Science Education Survey conducted by Horizon Research (2001). The selected schools represented various community settings (rural, small city, suburban and urban) and student population demographics. Once schools were selected, all mathematics teachers in the schools were invited to participate in the study. Across the eleven middle schools one teacher declined to participate, resulting in a sample that consisted of 51 teachers in the first year (grades 6 and7) and 66 teachers in the second year (grades 7 and 8) of the study.

Table 1: Mathematics textbooks used in schools participating in the two-year study.

Textbook Series	**Lead Author**	**Publisher**
Addison-Wesley Mathematics	Eicholz	Addison-Wesley Publishers
Connected Mathematics Project	Lappan	Prentice Hall
Houghton Mifflin Mathematics	Haubner	Houghton Mifflin
Math Advantage	Burton	Harcourt Brace & Company
Math Matters: An Integrated Approach	Lynch	South-Western Publishing Company
MATH Thematics	Billstein	McDougal Littell
Mathematics: Applications and Connections	Collins	Glencoe McGraw-Hill
Mathematics in Context	Romberg	Encyclopedia Britannica Educational Corporation
Middle School Math	Charles	Scott Foresman-Addison Wesley
Prentice Hall Mathematics	Charles	Pearson Prentice Hall
Saxon (Books: Math 65, 76, & 87)	Saxon	Saxon Publishing, Inc.

Four data gathering techniques were used to document teachers' use of their district-adopted textbook. These methods included classroom observations (three per year), Textbook-Use Diaries that teachers completed for three 10-day periods throughout the school year, and Table-of-Contents Diaries where teachers noted all lessons taught from the textbook. In addition, each teacher was interviewed one time per year to verify the information provided by the instruments and to understand the teachers' rationale for choices they made.

How often do teachers use the district-adopted textbook?

The Textbook-Use Diary documented the frequency of textbook usage by the teacher and students during the mathematics lesson and as a source of homework assignments. Frequencies were calculated as a percent of the total instructional days documented in the diaries. An overwhelming majority of teachers regularly used their district-adopted mathematics textbook during the recorded period. More specifically, 39% of the teachers used their textbook at least 90% of the documented instructional days, and over 70% used their textbook at least 3 out of every 4 instructional days.

Only one teacher in the sample reported using her textbook on less than half of the 30 instructional days.

How much of the textbook is presented throughout the school year?

Although most teachers reported frequent usage of their textbook, they did not necessarily follow their textbooks page-by-page. Table-of-Contents Diaries revealed the teachers taught an average of 65% of the lessons available in the textbook over the course of the school year. The percent of textbook lessons taught by teachers ranged from 25% to 97%.

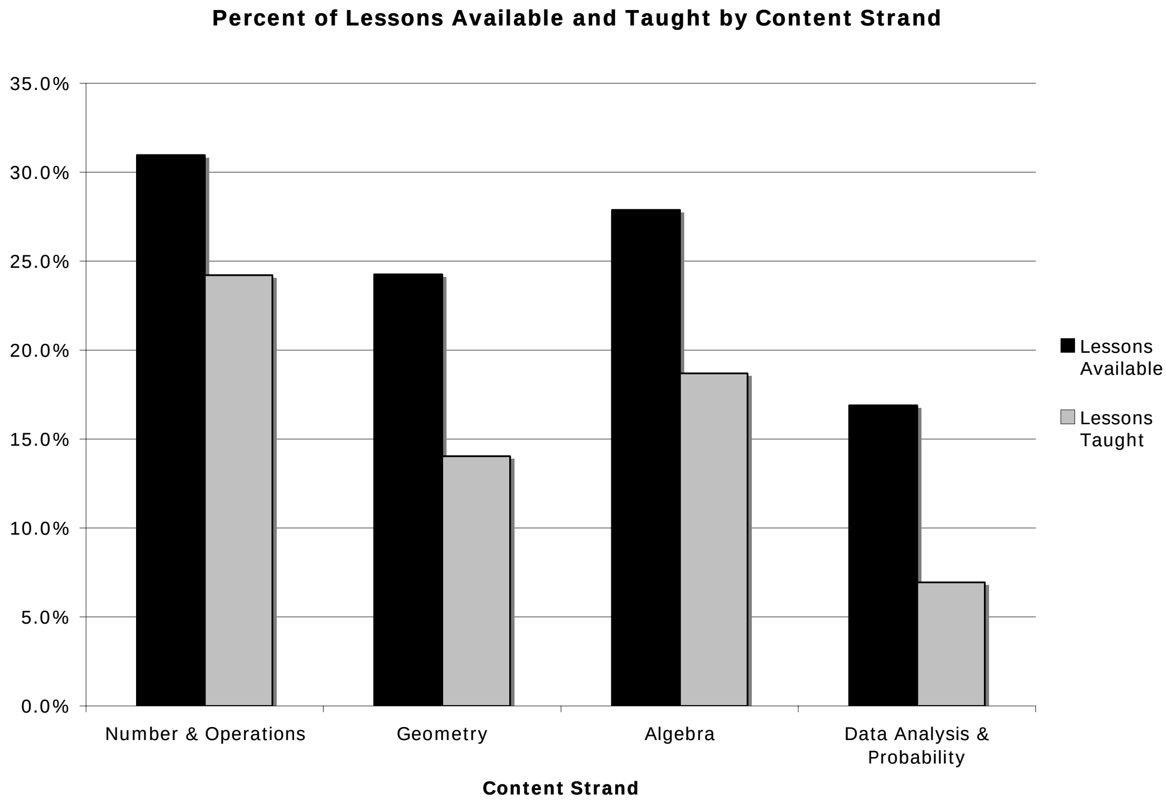
It was not uncommon for teachers in the same school using the same textbook to make different choices about their textbook. For example, two teachers in one school each reported using about 60% of their textbook lessons over the school year. However their selection of lessons was remarkably different. A first-year teacher taught *all* of the first 76 consecutive lessons in the textbook and *none* of its final 56 lessons. In contrast, a veteran teacher taught 71 of the textbook lessons but chose them throughout the entire sequence of the textbook, skipping some lessons but including at least a few lessons from every section of the textbook.

What mathematical content is emphasized over the school year?

The *written curriculum* (content presented within textbooks) was examined to determine the mathematical emphasis of middle school textbook lessons. Each textbook lesson was coded to indicate the content strand that was primarily emphasized: (a) Number & Operations, (b) Geometry & Measurement, (c) Algebra, and/or (d) Data Analysis & Probability. Across all textbooks, the largest number of lessons was devoted to Number & Operations (about 31%). Approximately 28% of the textbook lessons focused on Algebra and about 24% focused on Geometry & Measurement. Data Analysis and Probability accounted for the smallest portion of lessons - about 17%.

Given that most teachers in the study taught 60-70% of lessons in their textbook, they ostensibly made decisions regarding which lessons to teach and which lessons to omit. We analyzed the *enacted curriculum* (i.e., content from the textbook taught by teachers) to document the mathematics content emphasized in instruction. Teachers' decisions about what lessons to teach (and what lessons to skip) differed from the *written curriculum* with respect to the distribution of mathematics content. Based on the Table-of-Contents diaries, teachers reported implementing most of the lessons (about 78%) from the Number & Operations strand (see Figure 1). Algebra lessons were second most likely to be presented to students (about 67%) and Geometry lessons third (about 58%). Lessons related to Data Analysis & Probability were least likely to be presented to students (about 41%). Given that fewer lessons in Data Analysis & Probability were available than any other content strand, it is particularly notable that these lessons were also the least likely to be taught.

Figure 1: Percent of lessons available and implemented by teachers, sorted by content strand.



To what extent does the textbook influence instructional practices?

Classroom observations and teacher interviews were used to determine the influence of the district-adopted textbook on teachers' instructional practices. That is, to what extent do instructional practices mirror or reflect the lessons as presented in textbooks? Observers rendered judgments regarding the degree to which the textbook influenced the mathematical content and instructional strategies of the observed mathematics lesson. In cases where the mathematical focus and instructional activities in the observed lesson mirrored a textbook lesson, the observer noted that the textbook had "a great deal" of influence. On the other hand, if the lesson content focus and instructional activities were not drawn from a textbook lesson, then the observer recorded that the textbook did not influence the lesson. Table 2 reports the percent of observations noted in each category. It suggests that the textbook had a substantial influence on the mathematical content of the lesson and considerable, albeit less, influence on the instructional activity (presentation) of the lesson.

Table 2: Influence of textbook on lesson content and presentation, from classroom observations.

	Great deal of influence	Some influence	Very little influence	No influence
Mathematical Content Focus of Lesson	65.5%	27.8%	3.6%	3.2%
Instructional Activity (Presentation) of Lesson	44.8%	32.5%	16.3%	6.3%

Teacher interviews support the findings derived from the classroom observations. About 59% of teachers indicated that they rely primarily on the textbook when preparing and enacting lessons. The teachers indicated that the textbook serves as a main *source*, *guide* or an *outline* for planning and teaching their lesson. About 47% of teachers pointed out that their district-adopted textbook serves as a "scope and sequence" for their classes. One teacher said, "It provides the framework – what should be taught and how." Some teachers indicated that the textbook provides a comprehensive resource to follow, "It helps … gives an idea of the material to cover, something physical to use". One teacher referred to the textbook as a "roadmap." Likewise, another identified it as, "My bible. It is the basis for most of my instruction … source of homework, learning tool." The majority of teachers responded that the textbook determines "when [the order] and what" they teach.

In contrast, 38% of teachers specified that, even though they do agree that the textbook plays an important role in their teaching, they tend to rely more on their previous instructional experiences, knowledge and other supplemental resources.

These teacher responses did not suggest the “heavy” or “exclusive” dependence on the textbook.

Mathematics curriculum standards, both district and national, were mentioned by teachers as an important determinant of the mathematical focus of lessons. Indeed, about a fourth of the teachers indicated that lesson planning is driven by the local curriculum standards. Some teachers stated that they could rely heavily on their textbook because it is closely aligned with the district curriculum standards. Other teachers noted a lack of freedom to deviate from the textbook because district administrators provide guidelines and monitor the order and the amount of the textbook that must be presented to students.

Summary

The literature has long documented that U.S. teachers make substantial use of the district-adopted textbook to guide their lesson planning in mathematics classrooms. The study reported here confirms this overall finding. It also confirms that few teachers utilize the entire district-adopted textbook. In fact, most teachers, regardless of the particular textbook series, cover about 75% of the lessons included in their textbook.

Comparison of the *written* and enacted curricula confirms that teachers are more likely to give attention to some strands of mathematics (e.g., Number & Operation) than would be expected given the emphasis in their textbooks, while other mathematical strands (e.g., Data & Probability) are given less attention. That is, U.S. teachers are more likely to teach lessons in their textbook from the number and operation strand than lessons from the data and probability strand. Whether this reflects their official state or district curriculum standards or their own perception about what is important or what their students need is unclear. We also found that U.S. teachers are not necessarily bound by the placement of the topic in a textbook (i.e., Chapter 1 or Chapter 15), but tended to purposely choose lessons regardless of their placement in the textbook.

These data suggest that the district-adopted textbook strongly influences *what* mathematics is taught to middle school mathematics students as well as *how* students are engaged in learning mathematics. These data suggest that textbooks likely impact U.S. students' opportunity to learn mathematics in important ways.

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Investigations into an elementary school teacher's strategies of advancing children's mathematical thinking

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ABSTRACT

This is an interpretive study. The participating teacher "Chin" was a grade 1 teacher in a public elementary school in Taipei city. This co-operative intervention research lasted for one school year. The researcher played the roles of a facilitator and an investigator. We had weekly meetings to discuss her teaching. Chin progressed to the top level (Franke et al., 2001). Her action research showed that she had advanced the cognition levels of students. This paper documents how Chin advanced children's mathematical thinking. We focused on Chin's strategies of problem posing and teaching.

We found that Chin investigated children's thinking through conjecturing and experimenting. She posed problems and formed teaching strategies to detect, scaffold and advance children's mathematical thinking on the basis of her knowledge of children's cognition in mathematics. As a result, instructional reform does not succeed without sufficient knowledge of children's mathematics cognition. This study contributes to the plan of teacher education and reform-oriented teaching.

Key words: instructional strategies, mathematical thinking, teaching practice

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The curriculum standards issued by the Ministry of Education of Taiwan (1993) articulate a national vision for teaching mathematics. This vision includes engaging students in problem solving, mathematical connections, reasoning and communication, as the focuses of Standards of the United States (National Council of Teachers of Mathematics, NCTM, 2000). Calls for instructional reform in mathematics have been accompanied by demands for instructional changes. These changes require teachers to create classrooms that foster children's development of conceptual understanding of mathematics. The teacher's role shifts from problem solver to problem poser, as students shift from being imitators to problem solvers. However, many teachers can not enhance children's mathematical ability because they teach children various solution methods listed in the textbook or just ask them to present their own solution methods without establishing social and socialmathematical norms to develop the child's thinking (McClain & Cobb, 2001; Yackel & Cobb, 1996). Teachers should intervene in order to advance children's thinking (Fraivilling, Murphy, & Fuson, 1999). However, to date there has been insufficient articulation of particular strategies teachers can use to advance children's thinking in Taiwan. As a result, it is worth while revealing a teacher's effective strategies of advancing children's mathematical thinking. The purpose of this study was to explore how a teacher can advance the mathematical thinking of children.

CONCEPTUAL FRAMEWORK

The theoretical perspectives on classroom interactions stem from a constructivist view of knowing (Simon, 1995; von Glasersfeld, 1987) and the Vygotskian view of teaching as creating successive zones of proximal development (Vygotsky, 1978).

Central to Simon's Mathematics Teaching Cycle is the inherent tension between responding to the students' mathematics and creating purposeful pedagogy based on the teacher's goals for student learning (Simon, 1995, p. 76). Thus, teachers should play proactive roles in establishing social and socialmathematical norms to develop the way a child thinks (McClain & Cobb, 2001; Yackel & Cobb, 1996). While social norms concern the normative aspects of classroom actions and interactions, sociomathematical norms are specifically mathematical (Yackel & Cobb, 1996). These norms regulate classroom discourse. Examples of sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, and an efficient mathematical solution (Yackel & Cobb, 1996). These three norms involve a taken-as-shared sense. In contrast, another sociomathematical norm of what counts as an acceptable mathematical explanation and justification deals with the actual process of making a contribution (Yackel & Cobb, 1996).

Many studies (e.g. Franke, Carpenter, Levi, & Fennema's, 2001) provide evidence that knowledge of children's thinking has a powerful influence on teachers' instructional change. In Cognitively Guided Instruction (CGI) approach, teachers are encouraged to use research-based knowledge about children's mathematical thinking to make instructional decisions (Carpenter, Fennema, Franke, Levi, & Empson, 1999). CGI consists of research information about the development of a child's thinking. The problem-type frameworks emphasize semantic differences among problems and solution strategy hierarchies.

This case is an example of a teacher who taught in reform-oriented ways. She not only used research-based knowledge to make instructional decisions, but also focused on children's mathematical thinking. Mathematical thinking means the different ways a child thinks which belongs to different levels of mathematical conceptual development (Steffe, Cobb & von Glasersfeld, 1988). Take the composition and decomposition of two sets for instance. Count-all procedure (count each set separately then count the two together) belongs to sequential integration operations. Count-on procedure (count-on the number of elements in the second set, starting from the number in the first set) and count-back procedure (start from the larger number and count-back down the number sequence to find the number remaining) belong to progressive integration operations. The level of the second operation is higher than the level of the first one.

METHOD

This is an interpretive study. The participating teacher "Chin" was a grade 1 teacher in a public elementary school in Taipei city. There were 27 students in her class. This co-operative intervention research lasted for one school year. The researcher played three roles: participant observer, facilitator and investigator. We had weekly meetings to discuss Chin's teaching. The main activities engaged in the study were analyzing patterns of students' solutions and discussing literature, cases of mathematics teaching and individual problems encountered while teaching.

The data collected for this study included audio-taped interviews, video-taped classroom observations, audio-taped discussions, Chin's worksheets and reflective journals, the analysis of patterns of students' solutions, lesson plans, students' written work and an action research report. All video and audio-taped data were transcribed verbatim.

The data was analyzed using Cobb and Whitenack's (1996) methodological approach which can be used to analyze large sets of qualitative data. The teacher's teaching competence was analyzed according to Franke et al. (2001) 'Levels of Engagement with Children's Mathematical Thinking'. Multiple triangulation on the source, method, time and analyst were used to validate the data. Chin progressed to the top level (Franke et al., 2001). She had advanced her students' mathematical thinking since her action research showed that many students in sequential integration operations were promoted to progressive integration operations.

RESULTS AND DISCUSSION

First we describe two scenarios of Chin's typical teaching of promoting students' cognition, and then discuss her strategies of advancing children's mathematical thinking.

Scenarios of Chin's Teaching

Scenario 1

Chin posed problems as follows (problem posing, 05/01/2002):

1. *Johnny Bear is good at making pizza, while Andy Crocodile is an expert at making cake. They went to the market to sell pizza and cake together. Johnny made 76 dollars and Andy made 67 dollars in the morning. Who made more money? How do you know?*
2. *Andy Crocodile worked hard. He didn't rest and continued to work throughout the afternoon. However, Johnny Bear rested until 2 P.M. Finally, Andy made 32 dollars and Johnny made 21 dollars in the afternoon before they closed their stalls.*

a. How much did Andy make all day? (Solve it by jumping on the empty number line.)

Andy

*67*____

b. How much did Johnny make all day?

Johnny

*76*____

c. Who made more money?

Chin scaffolded students' learning according to their mathematical thinking. She encouraged students' various ways of problem solving, efficient solutions and formed sociomathematical norms to promote students' thinking at a more advanced level

(McClain & Cobb, 2001; Yackel & Cobb, 1996). She monitored whether individual student's solution strategy was advanced.

Posing problems to diagnose if students understand

The reason Chin posed the first problem was to diagnose whether students understood the place-value concept. She conjectured that those students who had no place-value concept would compare the unit place and think 7 is bigger than 6 so that 67 is bigger than 76.

Encouraging students' various ways of solutions and efficient solutions

Chin reflected on the students' solutions because the solutions were beyond her imagination as they solved the first problem. As a result, she encouraged students to use multiple solution methods. Chin monitored if low level students adopted the good methods of others. The student S3 solved the first problem by drawing 67 circles on the first line and 76 circles on the second line and then he matched them one by one. After discussing the efficiency of solutions, Chin wanted to make sure that S3 used a more effective strategy. Finally, the student found that S31's method was much better (see Figure. 1) and used this strategy at a latter time. Chin discovered that comparing different solutions and encouraging efficient solutions resulted in the elevation of students' strategies.

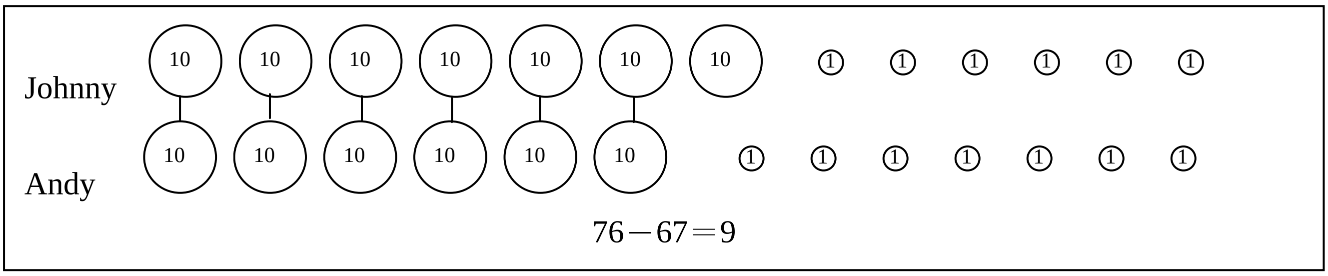

*Figure.*1. S31's solution method

Scaffolding students' learning according to their mathematical thinking

Chin used the characters and context of students' favorite story to pose the problems. She was concerned about the affective aspect of her students (Interview, 05/01/2002). She posed problems according to students' mathematical thinking to promote their cognition at a more advanced level. Chin would conjecture students' solution strategies and investigated the relationships between word problems and their solution methods. She found that students used the count-all procedure for a Change Add To problem, because the first given addend was small (Fuson, 1992). She conjectured that students would count on from the first given addend for a Change Add To problem, if she made the first addend bigger. The purpose of posing the second problem was to prevent students from using count-all strategies and to scaffold them to use count-on strategies by forcing them to jump on the empty number line (Discussion, 05/01/2002).

Monitoring if students' solution methods are advanced

Chin posed similar problems as a homework to detect if students adopted other more efficient methods (Interview, 05/01/2002). She found that many students' solutions were advanced. Chin monitored students' learning at all times.

Scenario 2

Chin posed the problems of "saving princess" structurally to make students aware that addition and subtraction can cancel each other out. She posed the first problem to facilitate students' effective strategy of count-back. The problems are as follows:

1. *Three brothers brought a treasure given by an old woman to visit the king. The king was very happy because he thought that the princess would be rescued. The king decided to dispatch a corps of soldiers to support them to save the princess in the Dark Valley. The three brothers departed with these soldiers. The total number of people going to save the princess was 51. How many soldiers did the king send off to support the three brothers?*
2. *A battle suddenly happened in their country as they were half way to save*

the princess. As a result, the king called back the soldiers. How many people were left to rescue the princess? (Worksheet, 06/11/2002)

Chin posed challenging problems according to students' mathematics cognition. She guided students to identify the same solutions and use efficient solutions, encouraged acceptable explanations, and promoted students' mathematics cognition at a more advanced level.

Identifying the same solutions

S29's and S33's solution strategies (See Figure.2 and Figure.3) were the same, but were different representations. Chin presented them to the students to identify.

Encouraging acceptable explanations

Chin was aware that S29's representation was too complex to understand for students (See Figure. 2). She invited S29 to explain. S29 connected the meaning of the problem and her solution by saying "Divide 51 into 50 and 1. The 1 becomes the first one of the three brothers, and then 50 minus 2 is the result." (Teaching, 06/11/2002) . Students who couldn't understand S29's representation could accept her representation after her explanation.

Chin arranged her teaching depending on students' solution strategies at different levels. After the whole class discussed solution methods of S11, S29, S33 and S34 (See Figure. 4), she asked students which strategy they understood and preferred. Consequently, just one student preferred S11's method which was drawing 51 circles and deleting 3 circles. 9 people adopted S34's method and 7 people adopted S29's and S33's methods. It means that most of the students didn't agree with the count-all strategy because it is inefficient. Otherwise, many students accepted the method of drawing coins. Chin was able to learn students' mathematical thinking by interacting with them. As a result, she could understand the more acceptable method and inspired students to use a more efficient method.

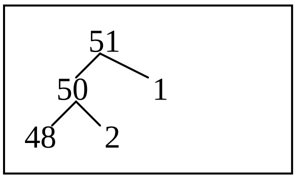

Figure.2. S29's solution method

51-1=50

50-2=48

Figure.3. S33's solution method

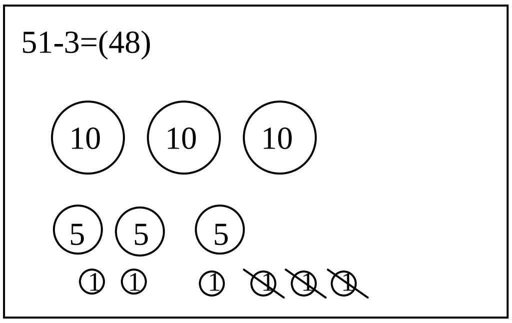

Figure.4. S34's solution method

Comparing efficient solutions with inefficient ones

Chin usually encouraged students to use effective and easy solutions (Teaching 06/12/2002). Although S28 wrote the number sentence 51-3, she used count-back strategy (start from 51, count-back down the number sequence 50, 49, 48 and find 48 remaining). S27 solved 3+(　)=51 by counting from 3 on. Chin compared S28's efficient method with S27's inefficient method. Children perceived that it took less time to solve the problem by using S28's strategy, while it took much more time to solve it by using S27's strategy. They appreciated the value of efficient solutions.

Reasoning and testing students' mathematical thinking

Chin concluded that we can't just identify students' mathematical thinking by their number sentences or representations. She often reasoned students' mathematical thinking and tested her inferences. For example, Chin found that a student wrote (48) + 3=51, and drew circles but not 48 circles. She conjectured that it is impossible to determine the answer to be 48 as the student had not finished drawing the circles. After asking the student, she found that the student used count-back strategy (Fuson, 1992). Chin always explored students' mathematical thinking by guiding them to record and explain their solution processes. She reasoned and tested their conceptual development levels.

Chin would judge the levels of students' solutions by interacting with them not only by using the mathematical conceptual development frameworks. Her knowledge of students' learning was not restricted to these frameworks. She hypothesized and tested students' cognition on the basis of these frameworks, and then refined them.

Discussing the structures of mathematics to promote students' cognition

It is helpful to advance students' thinking by reflecting on different types of problems (Bell, 1993). For example, addition and subtraction can cancel each other out. "Thinking backwards" requires the opposite operation to the one stated in the question. Chin discussed mathematical structures with students in order to promote their cognition at a more advanced levels. She related the problem to the number sentence by analyzing the problem in terms of parts and wholes. She explained that 51 people included 3 brothers by drawing diagrams and operating base-ten arithmetic blocks. The three brothers are part of the 51 people. When you have a part and a whole, you subtract to find the other part. She wanted to make students aware of the relationship between addition and subtraction. She asked students how to move blocks if the king had not dispatched the soldiers to help the three brothers. After students responded "remove base ten blocks of 48", Chin took away the blocks of 48 and represented the number sentence 51－48＝3 and then she put back the base ten blocks and wrote 3＋48＝51. After this process, students were aware that addition is the inverse operation of subtraction.

Chin's Strategies of Advancing Children's Mathematical Thinking

Chin's strategies of advancing students' mathematical thinking are elaborated in the following. The strategies of problem posing and teaching are discussed respectively.

As for the strategies of problem posing, Chin posed problems in order to understand, detect and promote students' mathematical thinking. She formed more challenging problems to elevate students' cognition based on their original solution methods. She conjectured students' solutions as she posed problems. She posed problems structurally according to students' cognition and explored the relationships between different problems and their solutions. She inferred why problems were difficult for students after teaching and revised them.

As for the strategies of teaching, Chin elicited students' mathematical thinking in order to probe their cognition. Chin taught students how to ask questions in order to understand others' thinking. She encouraged students to explain their methods and ask questions with each other. She patiently guided students to record and present solution methods. Chin conjectured and tested students' thinking.

Chin diagnosed and dealt with students' errors and helped students in problematic situation. She was interested in finding inefficient or wrong solutions. She attempted to probe why students could not understand subject material and explained how they learned by interacting with them. For example, Chin posed the problem "There are 6 strawberries in one dish. How many strawberries are there in 2 dishes? " to detect whether students wrote 6＋2. Chin posed "8 children played a game, and then came 4 children. How many children are there now? ". She observed the solution of every student carefully and found S6's answer was 11. She guessed that S6 counted 8, 9, 10, 11. Although S6 was reluctant to elaborate on his strategy, Chin encouraged S6 to explain with his fingers. Consequently, Chin guided S6 to expand on a correct counting method.

Chin modified her teaching according to students' thinking. She presented immediate reflection to adjust her instruction while teaching (Mcduffie, 2004). If she found that students did not understand, she would reflect on her teaching and solve the problem. Chin would conjecture the reason why students didn't understand and then experiment. For example, many students could present the number sentence 3=2+1 under the context of balance. Otherwise, students thought that it was wrong to write 3=2+1 as there was no context. She conjectured that students can not understand the meaning of equal sign without context. She experimented by posing a problem with another context.

Chin provided scaffolding on the basis of students' mathematical thinking. While discussing mathematics, Chin and her students formed sociomathematical norms. She guided students to identify the same solution methods, distinguish different methods, facilitate acceptable justification and adopted efficient solution methods. She provided students with the opportunity to discuss the mathematical structure to advance students' cognition.

Chin monitored the learning of her students. She would observe these students' solution strategies, diagnose if low level students understood and examine whether the students were progressing. Chin would record and analyze students' solution methods. As a result she knew every student's strategy very well. In addition, Chin Figured out new strategies to help her understand students' mathematical thinking. For instance, Chin would mark students' worksheets immediately after they completed them, circle the mistakes and circle the other place where the students must revise their solutions.

She didn't allow students to erase their errors in order to make parents know their children's thinking and misconceptions. Chin was able to capture each student's conceptual development process by observing his strategies at different time. She arranged her lessons according to students' thinking and advanced individual student's mathematics cognition through whole class discussion.

CONCLUSIONS

The frameworks of mathematical conceptual development provided a general guide to understand students' mathematical thinking. Chin posed problems and formed teaching strategies based on the frameworks to probe, understand and elevate students' thinking. She investigated students' mathematical thinking constantly. She revised and refined the knowledge of students' cognition through conjecturing and experimenting. Therefore, not only can we acquire the knowledge of mathematics from the view of quasi-empiricism (Lakatos, 1978), but we can also obtain the knowledge of students' thinking. Chin formed strategies of understanding and promoting students' mathematical thinking based on the knowledge of the students' cognition. As a result, instructional reform does not succeed without sufficient knowledge of a child's mathematics cognition. Therefore, this study contributes to the plan of teacher education and reform-oriented teaching.

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Investigating the Learning Expectations related to Grade 1-8 Geometry in Some Asian Countries and U.S. States

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Abstract

This study analyzes learning goals in grade 1-8 geometry across several U.S. states and high performing TIMSS Asian countries, including Singapore, Taiwan and Japan. In order to shorten this paper, only one topic-angle within the geometry strand was carefully analyzed.

Based on the official curriculum documents, results of this study indicate that the content, grade placement and cognitive level of learning expectations related to selected geometry topics vary markedly across documents.

To be precise, some documents such as the California and Achieve documents include more advanced topics within their 1-8 curriculum framework and introduce them earlier than the other documents. In addition, the Singapore document places comparably more emphasis on the topic of “angle” and less on “coordinate geometry” and “parallelism/perpendicularity.” Furthermore, the Missouri document emphasizes on “coordinate geometry” and “similarity” topics and de-emphasizes on the topics of “angle” and “parallelism/perpendicularity” than the other documents reviewed. In the meantime, considerable differences emerge across each topic. In sum, analyses from this study have provided a clear picture about the “angle” topic from each document.

Key Words: Curriculum Framework, Learning Expectation, NCLB, OTL, TIMSS.

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Introduction

Many international assessments support the view that many students in the United States are not learning the mathematics they need or are expected to learn, particularly when compared to peers in others countries (Beaton et al., 1996; Kenney & Silver, 1997; Mullis et al., 1997). Although the reasons for these differences are complex, educators generally agree that opportunity to learn (OTL) is a contributing, if not major, factor. Floden (2002) argued that "If OTL is not taken into account, its effect may be mistakenly attributed to some other attribute of the educational system."

Most educators acknowledge that various dimensions of the educational system, such as state or local policies, textbooks, classroom organization and teacher knowledge, have an influence on students' opportunity to learn and the quality of instruction students receive. As noted by authors of *The Underachieving Curriculum* (McKnight et al., 1987):

> "*One of the main functions of curriculum, as intended and as implemented, is to distribute the content of the curriculum throughout the days and years of schooling according to a coherent and reasoned set of goals*" (page 85).

The Third International Mathematics and Science Study (TIMSS) used a model called Potential Educational Experience (see Figure 1) to capture different aspects of how educational opportunities are shaped and how they are potentially related. In this model, curriculum goals at the system level represent the "intended curriculum." Historically, the intended curriculum has not been described in details within the United States.

International studies of mathematics and science achievement have consistently reported that students in Asian countries such as Singapore, Taiwan and Japan demonstrate higher levels of mathematics achievement than students in the United States. Especially, based on evidence from reports of TIMSS and the National

Assessment of Educational Progress (NAEP), geometry is an area of weakness for U.S. students (Kloosterman , 2004). However, little is known about how the curricula described in the documents differ from state or Asian country curriculum frameworks.

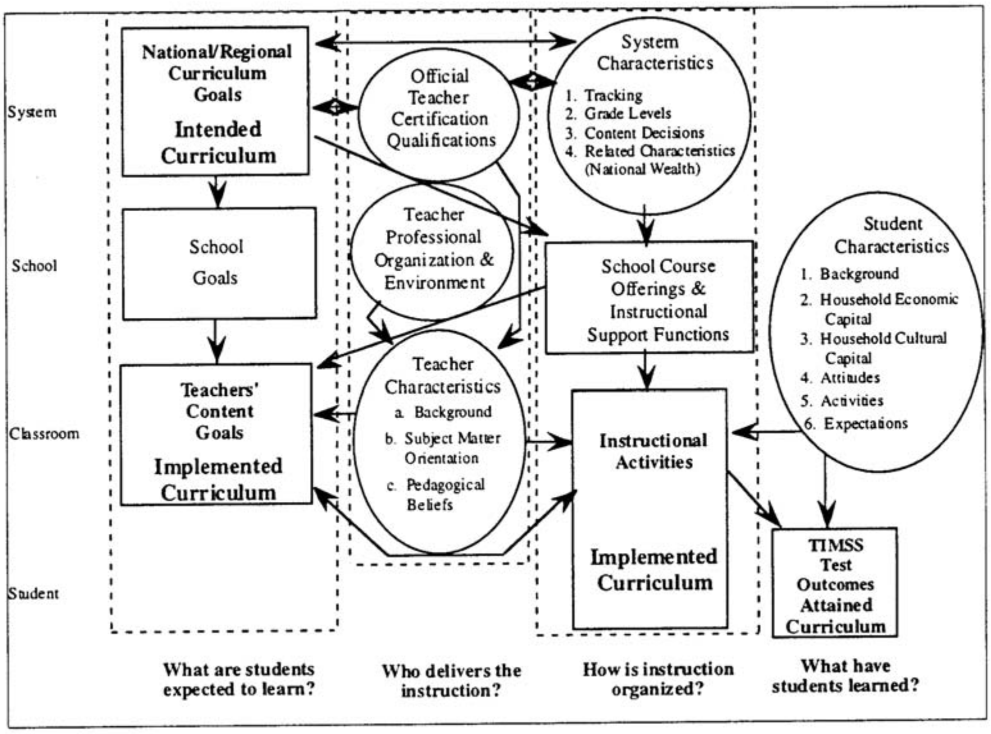

Figure1: The Model of Potential Educational Experiences.

(Source: Schmidt et al., 1997).

The passage of the No Child Left Behind (NCLB) legislation in 2002 prompted many states to identify grade-by-grade learning expectations (LEs) and align these expectations to mandated state annual assessments in grades 3-8. In fact, as of June 2005, at least 44 states have official curriculum documents that specify grade-by-grade learning expectations in mathematics (Reys et al., 2005).

More recently, Achieve, Inc., an independent nonprofit organization created by governors and corporate leaders to help raise standards and performance in American schools, released a grade-by-grade curriculum framework for K-8 (Achieve, Inc., 2004). The Achieve document was also reviewed for this study as it represents a national proposal including grade-by-grade learning expectations, organized by strand, from a U.S. organizational group. In fact, special attention in this study is placed on the following research question:

To what extent and in what ways are learning expectations associated with geometry strand similar or different in emphasis and grade placement in selected Asian countries and U.S. states as described in the official standards of mathematics curriculum documents?

This analysis may partially explain differences in performance among students in these countries and states, particularly if the intended curriculum is an important contributor to what students have an opportunity to learn.

Methodology

The selection of countries for this study was based on the performance on the TIMSS assessment. The selection of U.S. states was based on student performance on the NAEP-2000 assessment and on the evaluation of official state curriculum documents by the Fordham Foundation. Geometry strand was selected for analysis because U.S. students perform relatively poorly on items related to this strand, compared to students in other countries and compared to their performance on other strands of mathematics.

A coding system was developed which consisted of the general categories: *Action, Object, Cognitive Domain (SEC), and Tools*. For each learning expectation in the geometry strand of the curriculum documents, the following information was coded:

- Object-the main noun(s) in the learning expectation
- Action-the main verb(s) in the learning expectation
- Tools-equipment specified for use within the learning expectation
- Cognitive Domain-identification of cognitive level of learning expectation

based on the *Survey of Enacted Curriculum* protocol (CCSSO, 1999).

Table 1 provides a sample of how learning expectations were coded in this study.

Table 1: Sample of coded learning expectations

Learning Expectation	Grade	Action	Object	Cog. Demand	Tools
To understand the meaning of units and measurement of volume. (Japan)	6	Understand	Units and measurement of volume	Level 3	--
Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle. (California, grade 3).	3	Identify, determine	Right angles	Level 1	--
Pupils can understand the meaning that two triangles are congruent through construction with straightedge and compass. (Taiwan)	8	Understand	Congruent triangles	Level 3	Straightedge/compass
Student will identify congruent and similar figures. (Minnesota)	4	Identify	Congruent and similar figures	Level 1	--
Students can use formulas routinely for finding the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders. (CA)	7	Use formulas	Surface area and volume	Level 2	--
Solve problems involving surface areas and/or volume	8	Solve problems	Volume and surface area	Level 1,4	--

of a rectangular or triangular prism, or cylinder. (Missouri)					
Find the volumes and surface areas of cubes, cuboids, prisms and cylinders. (Singapore)	7	Find	Volume and surface area	Level 2	

Analysis of the Learning Expectations in Geometric Topics

Recall that the general strategy for analysis was based on the "topic tracing" method developed by TIMSS researchers. That is, for each topic, all LEs related to that topic within each curriculum document (3 countries-Singapore, Taiwan, Japan; 3 states-Minnesota, Missouri, California; Achieve) were identified and the following information was compiled:

- A description of the focus of the topic by grade level and document.
- The grade where the topic is intended to be first introduced to students.
- The range of grades during which instruction was intended to take place on the topic.
- Any grade for which the topic was to be a special emphasis.

The main goal of this analysis was to describe the focus of specific content, depth of coverage, and grade placement where particular LEs were emphasized.

<u>Summary of Learning Expectations Related to "Angle"</u>

The concept of "Angle" was one of topics analyzed within the geometry strand. Based on the analysis, a summary of the content emphasis and grade placement for this topic was provided. In addition, similarities and differences in emphasis and grade placement across the seven documents were noted.

In all 73 learning expectations related to angle were identified across the seven documents. The earliest LE appears in grade 2 of the Taiwan document and states:

Pupils can recognize an angle, straight line, and plane (containing solid shapes) in their daily life. (Taiwan, grade 2)

Other early grade angle LEs include:

Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle. (California, grade 3).

Student will identify, describe, and classify two-dimensional shapes according to number and length of sides and kinds of angles. (Minnesota, grade 3).

Recognize that angles on a straight line add up to 180 degrees and that angles around a point add up to 360 degrees. (Achieve, grade 5)

A sample of Grade 7 and 8 LEs related to angles includes:

To determine and verify logically the relationship between inscribed angles and central angles through observation and experimentation. (Japan, grade 8).

Describe relationships between corresponding sides, corresponding angles, and corresponding perimeters of similar polygons. (Missouri, grade 7).

Pupils should be able to calculate the sum of interior angles of a polygon and the sum of exterior angles of a polygon. (Singapore, grade 8).

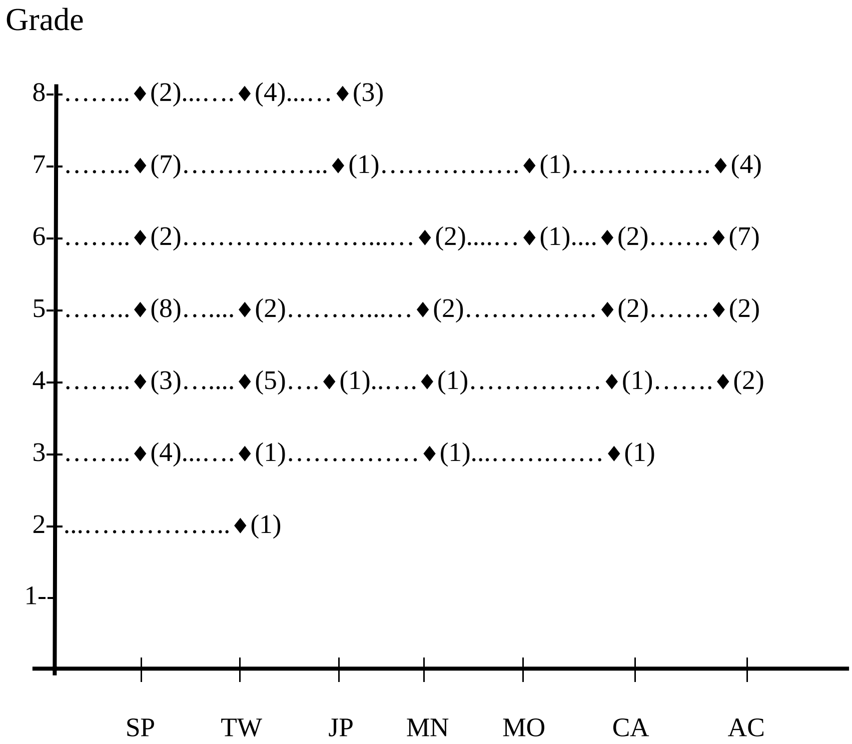

Figure 2 : Number and grade placement of learning expectations related to “Angle” within Geometry strand.

(Remark: The number inside parentheses indicates the number of learning expectations).

Common learning expectations related to angle include: identifying and/or drawing special angles (e.g., right, straight, acute, obtuse, complimentary), comparing angles, drawing and measuring angles (with and without tools), understanding relationships among particular angles (e.g., vertical angles or angles formed by parallel lines intersected by a third line), finding unknown angles based on properties (e.g., sum of angles of triangle), classifying polygons based on size and relationship of angles, constructing angle bisectors, etc.

Six common learning goals were noted within the set of LEs (see Table 2). For example, in seven of the documents, students are expected to identify angles, including special angles such as a right angle and this expectation occurs at grade 3 or 4 across these seven documents. On the other hand, the LE to draw or measure an angle with a protractor, while common to seven documents, occurs at grade four in

Singapore and Taiwan, grade 5 in California, but at grade 6 in Minnesota and Achieve, and grade 7 in Japan. Note that the learning expectations in Missouri are kind of addressed in generalizations such as "Classify 2-D and 3-D shapes based on their properties" or "Analyze characteristics and properties of 2-D and 3-D geometric shapes." Only two LEs in Missouri are specifically related to the angle, that is, to describe the relationships of similar triangles and similar polygons.

Table 2 : Common learning expectations related to the "Angle" topic

Common Learning Expectation	SP	TW	JP	MN	MO	CA	AC
Identify angle and right angle	G3	G3, G4	G4	G4	G4*	G3	G4
Draw or measure an angle using the protractor.	G4	G4	G7	G6	G7*	G5	G6
Understand the sum of the interior angles in a triangle	G5	G5	G8	G5	-	G5	G6
Find unknown angle involving some basic properties	G5-G7	G8	G8	G6	-	G6	G6
Understand all angles' relationships when two parallel lines are intersected by another line	G7	G8	G8	G6	-	G6	G7
Calculate the angles or solve the problems related to a quadrilateral or polygon	G8	G8	G8	G5	-	G5	G6

Remarks: 1. G3 means the learning expectation is provided for Grade 3.
2. "*" indicates this LE is found in Measurement strand.
3. "-"indicates no specific statement in the LEs.

Among the 73 LEs some were noted only within one or two documents. For example, the Singapore document includes the following LEs not found in other documents:

To estimate the size of angle (Singapore, grade 4).

To recognize the exterior angle of a triangle is equal to the sum of the interior *opposite angles (Singapore, grade 5).*

To identify the vertically opposite angles, and to recognize they are equal (Singapore, grade 5).

Likewise, the Taiwan, Japan, Missouri and Achieve documents include LEs not found in other documents. These include:

To understand the relationships between side and angle in a triangle (Taiwan, grade 8).

To determine and verify the relationships between inscribed angles and central angles (Japan, grade 8).

The rigid motion doesn't change the length and angle of polygonal figures (Achieve, grade 6).

Criteria such as SSS, SAS and AA for similar triangles are addressed (Achieve, grade 7).

Based on the analyses of the collected documents, Table 3 summarizes the grade at which the topic of angle receives special emphasis. "Special emphasis" indicates that the common learning expectations of this topic are addressed and that a substantial amount of time (in proportion to other topics from geometry) is devoted to angle. In general, attention to this topic is concentrated in Grades 4 - 8.

Table 3 : Grades for special emphasis on "Angle" topic

	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Singapore					√		√	
Taiwan				√				√
Japan								√
Minnesota					√	√		
Missouri					√	√		
California					√	√		

Achieve						√	√	

<u>Summary of Emphasis on Angle by Country/State</u>

As noted in Figure 2, the Singapore document includes the largest number of LEs related to angle (26 in all) with the greatest emphasis in grades 5 and 7. Students in Grade 3 and Grade 4 identify angles in 2-D shapes and draw or measure angles using a protractor. Using properties of geometric figures to find unknown angles is an important theme among the set of LEs, particularly from Grade 5 through 7. In addition, students in Grade 7 are expected to use tools to construct angle bisectors and to measure angles and students in Grade 8 are expected to calculate the sum of angles or unknown angles related to a polygon.

In contrast, Taiwan has fewer LEs related to angle. Most of these learning expectations emphasize understanding the meaning of "angle," "right angle," "rotating an angle," "unit degree," "180 degrees," and "360 degrees." In other words, these LEs related to basic terms are mostly categorized into the low levels in cognitive domain.

Minnesota has a very condensed set of LEs related to angle (6 LEs spanning grades 3-6). In Grade 3 and Grade 4, students classify 2-D shapes based on their angles and identify right angles. In Grade 5 and Grade 6, students focus on the sum of the angles in triangles and quadrilaterals. In addition, students solve real world problems using knowledge of angles, and learn to measure or draw the angles using common tools such as ruler, compass, protractor or software.

In Japan, the main emphasis for this topic is in Grades 7 and Grade 8. Students in Grade 7 will learn and use the basic methods to draw the angle bisectors. In Grade 8, emphasis is on the properties of parallel lines and angles and properties of the angles of polygons. Through observation and experimentation, students determine and verify the relationship between inscribed angles and central angles.

Only two learning expectations on this topic were identified in the Missouri document. One at grade 6 and the other at grade 7. In both cases, the focus is on relationships between corresponding angles and the length of corresponding sides (for similar triangles in grade 6 and similar polygons in grade 7).

The primary emphasis on angles in the California document is at grades 5 and 6. Students in Grade 5 are expected to measure and draw angles and know the sum of the interior angles of triangles and quadrilaterals. Students in Grade 6 are expected to solve problems using some properties of angles.

In the Achieve document, LEs related to angle appear in Grade 4 through Grade 7. In Grade 4 and Grade 5, students identify acute, obtuse and right angles. They also focus on the degree as a unit of measure and gain facility in measuring angles using degree as the unit. Students in Grade 6 have the opportunity to learn some critical properties of transformation, congruent triangles and symmetric figures. Finally, students in Grade 7 recognize equal angles, such as corresponding angles and alternate interior angles, as well as several criteria of similar triangles.

Weight of Topic Within Geometry Strand

In order to gauge the relative emphasis (weight) of angle within the Geometry strand, Table 4 provides a summary of the number of learning expectation associated with angle, and the percent with respect to the total number of LEs within the Geometry strand.

Table 4 : The Weight of topic-Angle.

	SP	TW	JP	MN	MO	CA	AC
Number of LEs	26	13	5	6	2	6	15
Percent of	33.3%	17.6%	19.2%	15.8%	4.1%	*	13.0%

Total Geometric LEs							

Remark: The CA document includes Geometry & Measurement together.

This table indicates that Singapore has the highest percentage related to this topic within the Geometry strand. By contrast, Missouri has relatively low weight. In general, the Asian countries have higher weight than the U.S. states in this topic.

Cognitive Level of Learning Expectations Related to Angle

Recall that the cognitive level for each learning expectation was coded using the Survey of Enacted Curriculum (SEC) protocol (CCSSO, 1999). Table 5 provides a summary of the distribution of levels in cognitive demand. Note that each LE may be coded into double levels.

Table 5 : Number and Distribution of Level in Cognitive Domain for LEs related to Angle

SEC / Country/State	N of LEs	Memorize Fact, Def. &Formula	Perform Procedures	Demonstrate Understanding	Conjecture, Generalize & Prove	Solve Problems, Connect
Singapore	26	19%	62%	19%	-	-
Taiwan	13	39%	8%	46%	-	8%
Japan	5	40%	20%	60%	40%	-
Minnesota	6	50%	17%	33%	-	17%
Missouri	2	-	-	100%	-	-
California	6	67%	17%	-	-	33%
Achieve	15	40%	33%	27%	-	-

Table 5 has provided evidence that most LEs under this topic are categorized into the first three levels of cognitive domain. Particularly enough, all countries/states except Japan have no learning expectations categorized into the higher level of “Conjecture, Generalize & Prove” in the SEC cognitive domain.

<u>Reference to Use of Tools</u>

Within the set of angle LEs, reference is made to a variety of tools including: ruler, set-squares, protractors, compasses, straightedge, and software. Although no specific tools about this topic are mentioned in the Japanese document, drawing figures involving bisecting angles are addressed. Also, Missouri made no mention of a tool specification for this topic.

Concluding Remarks

Learning expectations (LEs) represent what students are expected to learn. Based on the data analyses, researcher realizes that some content similarities and differences of LEs related to "angle" do exist in different countries/states. Accordingly, these differences raise the "opportunity to learn" issue which may also influence student achievement.

This examination revealed that all Asian countries have higher "angle" weights within the Geometry strand than the U.S. states. Meanwhile, under this topic, many LEs in ACHIEVE, and the states of California and Minnesota are categorized into lower level "Memorize fact, definition, formula" of cognitive domain. Comparably, the majority of LEs in Japan are categorized into higher levels of cognitive domain. These results may suggest that high level LEs that favor conceptual understanding have a close link and distinct benefits for student achievement. All these findings are perhaps worthy of our attention.

To conclude, "curriculum" is an important channel of influence to consider in understanding differences in student performance on measures such as TIMSS. Analyses from this study have provided indicators to understand the "Angle" topic from different countries/states, including the range of grades during which instruction was intended to take place on the topic, the "Angle" weights within the Geometry strand, the grades of special emphasis on this topic, and grade by grade cognitive levels from SEC. All these indicators, together with the common learning expectations listed in Table 2, mostly provide a broad profile of the attention given to angle across these countries and states. Future research may pursue the impact of curriculum documents on teacher practice.

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Perspective of prospective pedagogy in early Algebra: US-Russian Forum on Elementary Mathematics and Measure Up Curriculum

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Abstract

Algebra is an abstract concept for younger grades. If students could successfully learn in algebra, that might lead students to successful completion of advance mathematics. This research is to give an overview of the US-Russian and measure up curriculum and how it can be implemented to help students build a solid foundation in elementary mathematics.

Key words: Algebra, Elementary Mathematics, and Mathematic Curriculum.

Introduction

Traditional Elementary algebra teaching operations are performed on symbols rather than numbers, because algebra is defined as a generalization and abstraction of arithmetic. But teachers can teach students who have no knowledge of mathematics beyond the basic principles of arithmetic. Successful work in the former Soviet Union Davydov (1991) and Bodanskii, (1991) found that even younger children are capable of learning algebra. Their approach is to introduce algebra in the early grades and focus on the concept of function. They identified previously overlooked opportunities to explore the algebraic character of early mathematics. If elementary teachers could give questions about algebra problems and find a forum to help students to solve mathematic problems with lots of clear explanations and helpful hints, then, students can have plenty of hints and shortcuts for working with unknown quantities. Teaching algebra to children would not late , as Schlieman's results suggest that 3rd and 4th grade students could learn and understand elementary algebraic ideas (Schliemann, Carraher, Brizuela, Earnest, Goodrow, Lara-Roth. & Peled, 2003).

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There is very little algebraic character in the elementary educational curriculum of Taiwan, especially in first and second grade. Currently, in the Taiwan Curriculum standards /guidelines we are trying to organize some units to teach algebra in the elementary school, but they are still not in effect. Because elementary students' cognitive development is in a preoperational period, algebra functions as a language system for ideas within mathematics and it is abstract for elementary students. In the younger grades, things are more difficult in algebra class if the teachers did not prepare the students to learn early, or do not know what kind strategies to use in teaching algebra for students in the younger grades to be able to connect as middle high students in the future. How do teachers make it through algebra in teaching first and second graders? US-Russian Working Forum and Measure up Curriculum might give an example to work it out. Hence, the purpose of research is to explore an alternative curriculum to help young students to learn algebra in elementary school. What's more the alternative curriculum proposes pedagogy and materials to guide students to learn algebra in elementary school and keep students from failing to connect in middle high school.

An alternative mathematical curriculum overview

US-Russian working forum and measure up curriculum (E-D & MU curriculum) is the Elkonin-Davydov curriculum -- a model for the US in elementary mathematics – by the Curriculum Research and Development Group (CRDG) at the Univerisity of Hawaii. The foundation of the curriculum and development is based on the work of a group of Russian psychologists, mathematicians, and educators. The objective of E-D & MU curriculum is to develop children's algebraic thinking, conceptualization, and skills in grades 1-5.

The features of the E-D & MU curriculum emphasize the teaching of generalizations before the teaching of specific cases or examples. The curriculum has been applied in the UH Lab School for five years. In the E-D & MU curriculum, first grade children start with direct comparisons of continuous quantities (length, width, area, mass, and volume) and the use of symbols to describe the relation of compared quantities as equal, greater than, and less than. Second grade children do addition and subtraction that are introduced as action with continuous quantities (like water or grain), without using numbers. The computation through actions with objects directly parallels and models the mathematical content of the operations, which children also learn to represent through the use of other objects, graphical constructions (like line segments), symbols and letters.

Again, Measure Up is designed to approach an algebra focus in elementary school mathematics using measurement as its principal context to help all students successfully complete an algebra course in the future. In order to reach this purpose, CRDG planned a project that focuses on creating elementary mathematics materials for students and teachers employing results from Elkonin-Davydov research, designing a grades 1-5 mathematics curriculum integrating strategies appropriate to children's developmental levels and developing a classroom assessment system for grades 1-5.

A glance at the research context

The University of Hawaii Lab School applied the US-Russian and measure up curriculum in their elementary mathematics from first to fifth grade. The Project committee and staff were involved in restructuring the early research and preliminary materials from Russia. They also designed original materials that are suitable for US students. These materials blend the early research work and the new findings from research conducted during the development phrase. As for the project committee, they are all professors. The leader of the committee had visited Russia with her group members. The committee, except CRDG, is from the institute of Developmental Psychology and Best practices in Education at the University of Hawaii.

The Project staff is made up of seven mathematic teachers. The director of the staff is a UH professor and leads the staff to plan and improve the teaching. The staff has seven members and works in a teaching office. All staff has to teach mathematics, including the director. They always take turns in teaching elementary mathematics to each grade. The head teacher of the staff teaches kindergarten, while the first graders have math class, as well as teaches second grade and third grade mathematics. Obviously, the curriculum needs to recruit more teachers to support the teaching. So, the first grade and second grade teachers come from Doe school, but they are new teachers in the Lab School. Kindergarteners and first graders mix together; second graders and graders mix together to learn all subjects except mathematics. Recently, CRDC would like to disseminate materials and a professional development course nationally. In spite of the

UH Lab School implementing the curriculum, there is another connecting school in Hawaii.

A class of ten students has 45-minute lessons five days a week. The students come from diverse ethnic and socio-economic background. Ten students are selected from five different ethnic groups: two Japanese, two Philippines, two Caucasian, two Hawaiian, and two others who are all represented in the population.

The feature of E-D & MU curriculum

A significant factor of the E-D & MU curriculum project which is different from other elementary mathematics projects is to develop mathematical understanding. From grade 1, students start comparing attributes using continuous quantities. They describe the relationship between these quantities using direct and indirect measurement. But, every mathematical idea developed is connected to measurement. Here are a few of the critical themes of each grade:

Grade 1- symbolic representation of quantities, concept of unit, addition and subtraction;

Grade 2- conceptual understanding of place value and operation with multi-digit number;

Grade 3- multiplication, division, and basic mathematical properties;

Grade 4- transformational geometry, geometric measurement and numbers less than one (fractions, decimals);

Grade 5- is currently in development.

Teaching

In the classroom, teachers adopt multiple instructional strategies, like whole-class demonstrations, guide hands-on lab experiences, individual work period, student discussions, and stations. For example (Supplementary Documents for lesson sample):

The first grade teacher taught the students and applied the transitive property of equality by asking them to create a story that would match the given statements.

$$\begin{array}{c} M = E \\ \underline{P = E} \\ M = P \end{array}$$

The teacher asked the students to make up their own stories to go with these statements.

Today we are going to be storytellers! I will give you some statements like these. First read

the statements. Complete the statement below the line, about the quantities that were indirectly compared. After you complete the statement, write a story about the statements. Your story should tell what attribute was compared, and how the quantities compared. After you write the story you may draw and color a picture to go with it.

When the students had done that, the students shared their stories and hung them up for all to enjoy.

In another case, the first grade teacher asked the children to use the 'The inequality Symbols'.

Which Volume is greater? How do you know? The teacher showed > and < symbols to the students.

How do the volume container T and volume container S compare? How do you know? Or, how did you decide?

The volume in T and S is not equal. The teacher asked the students to record a statement about the quantities using the not equal to ($\neq$) and greater than (>) signs.

We read this as volume T is greater than volume S. Record this in your notebooks.

T > S.

The second grade teacher taught the students with multiple unit-measures to create a quantity. Quantities are recorded as equations, arrow notations, part-whole diagrams and line segments.

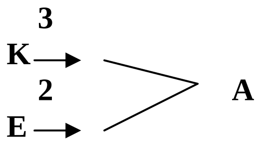
This is length A. She showed students the two unit-lengths K and E.

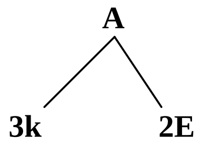
If we measured length A with length-unit K, and we measured length A again with length- unit E, which length-unit would we have to use more times? Let's measure length A. Which length-unit would take less time to measure length A?

Had a student use length-**unit** E to complete the measurement**.**

Let's represent length A by drawing a line segment on the board.

Because length A is made up of three length-unit K and two length- unit E, there are really two parts that make up length A. What other ways can we record the information from length A? Have students record other representations (arrow notation, part-whole diagram, equation) and discuss as needed.





3K+2E=A

Assessment

Students are assessed throughout the program. Students' class work, homework, participation in class, written assignments, and oral and written tests give teachers information related to each child's progress. However, the Lab School students will attend a state test in the third grade. Homework is one part of the assessment. It is assigned, occasionally in grades1-3, daily by grades 4 and 5. The attached 'tips' offer suggestions for how parents can support their children when they have homework.

Discussion

When looking at the CRDG make efforts to implement the E-D & MU curriculum, the curriculum would shed light on the learning of algebra in elementary grades. The curriculum implicates some thoughts in elementary mathematics teaching. Such as, in which grade can algebra be taught? Could it be taught in the younger grades? What is the key concept of algebra should be taught in elementary school? Does it work to incorporate other subjects in teaching algebra?

First, The E-D& MU curriculum proposes a striking opinion that children can learn in algebraic context early. Students begin early by building on informal knowledge in context. First-grade children can start with comparisons of continuous quantities and use symbols to describe the relation of compared quantities. Likewise, the E-D & MU curriculum develops proficiency in algebra which provides students the opportunity to learn with everyday life concrete objects to replace abstract symbols.

Second, algebra taught in elementary school is difficult. One factor is a misconception of the meaning of the equal sign. It often is thought to be an operator symbol by students, and also to be a cue to do something, instead of an expression of relationship (Falkner, Levi, & Carpenter, 1999; Filloy, & Rojano,1989). The curriculum could clarify and deliver the key concept for students to resolve the algebraic misconception and make a new way to learn algebra in elementary grades.

Third, Kaput (2001) suggests that teaching algebra can be integrated into other subject matter. The project committee and staff create elementary mathematics materials for teachers to instruct algebra in a diversity of ways. Teachers apply story telling, hands-on lab, and station to recall doing algebraic problem solving. These actions are based on the rationale that is a reference to a Vygotskiian (Elkonin-Davydov) distinction between spontaneous and scientific concepts. Spontaneous or empirical concepts were developed when students could abstract properties from concrete experience or instances. Dealing with an unknown quantity is to teach with objects in the early grades, so that students learn basic function to help them excel at math.

Besides, students solve algebraic sequenced measurement problems with equal and unequal concepts as a result of many months of teaching which permits children to discover that various types of real numbers and their operations are interconnected. These connections would give children a strong foundation for advanced studies.

Conclusion

Since CRDG has undertaken the E-D & MU curriculum of development and research for five years, the group endeavors to put theory into practice. Their efforts inspire educators to greater efforts of innovation in curriculum and teaching. The implementation of E-D & MU curriculum not only is an example for innovation curriculum in Taiwan but also might verify that children could learn early through an algebraic context and achieve better middle school and high school mathematics grades. So, through the E-D & MU curriculum, algebraic notation and concepts can be introduced from the very beginning of mathematics instruction, even though the curriculum is an ongoing project.

After over viewing the E-D & MU curriculum, some suggestions might be considered for the curriculum. The key factor which influences the implementation of results is professional teachers. Right now the project staff undertakes to develop material and teaching. They need more expert teachers to be involved in the project. It is necessary to create a professional development program for pre-service teachers.

Equally important, in having ten students in a class, the amount might not make competition for students in a small group. Moreover, the fact that only a few students are used in the experiment to test the theory of E-D& MU curriculum might make it difficult to be generalized in the future. Much more can be achieved if children participate in early algebra activities on a daily basis, as part of their regular curriculum.

In brief, Taiwan is strong on algebra learning in late elementary school, but E-D & MU curriculum teaches it from the younger grades and builds as it goes on to each of the grades. From the curriculum's experience, it would encourage the innovation of Taiwan curriculum of elementary mathematics to act to build a solid foundation for elementary students in algebra.

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NEWS

（一）2006/06/30~2006/07/05

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（二）2006/07/16~2006/07/21

Psychology of Mathematics Education30

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主編室

稿 約

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（一）本刊以徵選實務性的數學教育刊物為主，舉凡任何數學創新教學之方法或策略、數學教學實務經驗、數學課程設計與實踐之心得分享等皆為本刊之首要選擇標的；

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（2）圖書單行本，請依下列順序書寫：作者、出版年（西元)、書名、版次、出版地、出版社、頁數。

例：張春興（1996)。*教育心理學*。台北：東華。頁64-104。

8. 稿件順序為：首頁資料（題目、作者真實姓名及服務機關、通訊地址及電話；若需以筆名發表，請註明)、中文摘要、正文（包括參考文獻或註釋)、末頁資料（以英文書明題目、作者姓名及服務機關、並附英文摘要）及圖表（編號須與正文中之編號一致)。

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