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An Exploratory Study on Influences of a Mathematical Culture Course on University Students' Mathematics Beliefs – the Case in a Medical University

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In this paper, we present an exploratory study on how a liberal-arts course about the culture and history of mathematics influenced the mathematics beliefs of medical university students in Taiwan. This study used a single-group pretest-posttest design. The research tools of this study included: (1) a liberal-arts mathematics course with an emphasis on history and culture, and (2) a 20-question Likert-scale questionnaire used in the pre-test and the post-test. The questions were separated into two dimensions, aiming to investigate students' beliefs about the nature and values of mathematics. A total of 100 students took the pre-test, participated in the teaching experiment, and finally took the post-test. In the teaching experiment course, titled "Mathematical Thinking in the Multicultural Contexts", students were exposed to mathematical topics presented in their historical contexts. There were also examples of distinct approaches to similar problems by scholars in different civilisations, such as comparing Liu Hui's work and Euclid's *Elements*. Students were also required to make artistic creations related to mathematics. The results showed that part of the students' beliefs did change. In the dimension of the nature of mathematics, after taking the course, the students were more prone to believe that "generalisation" was a method of thinking in mathematics; however, the results also revealed that the course did not clarify the difference between the "context of justification" and the "context of discovery" for students. As for the values of mathematics, students were more prone to believe that "sensibility to beauty" and "creativity" were important values of mathematics.

Keywords: HPM, liberal arts, mathematics beliefs, medical university students.

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數學文化通識課程對大學生數學信念之影響初探—以醫學大學為例

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本篇論文旨在描述作者對於一數學文化通識課程如何影響醫學大學學生數學信念之初探。本篇論文敘述的研究使用單一群體前後測的實驗設計，其研究工具包含（1）一門關於數學史與數學文化的大學數學通識課程，以及（2）一份包含 20 個問題的 Likert-scale 數學信念問卷。問卷的內容包含兩個向度：數學本質與數學價值。共有 100 位同學修課並同時參與前後測。教學實驗進行的課程名稱為「多元文化中的數學思維」。課程中學生會接觸到不同文化與歷史場景中的數學知識。學生也會看到古文明中對類似問題的相異解法，例如比較東亞的劉徽對錐體體積的研究與古希臘歐幾理德《幾何原本》中相關命題的證明。學生在課程中也被要求將數學元素融入他們的藝術創作作業中。研究結果顯示，學生部分的數學信念確實有改變。在數學本質向度上，學生更傾向同意「一般化」是數學思考的方法之一。然而，結果也顯示這門課程並沒有幫學生釐清「核證的脈絡」與「發現的脈絡」。至於在數學價值的向度上，學生更傾向同意「數學培養創造力」，以及「數學培養美感」這兩項的價值。

關鍵詞：數學史與數學教學、通識教育、數學信念、醫學大學學生

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I. Introduction

In Taiwan, medical practitioners—physicians, dentists, pharmacists, etc—usually have high socio-economic statuses, and are generally well-respected in Taiwan. The reason, beside their above-average income, lies in history. In Japanese colonial times, before WWII, Taiwanese people were rarely allowed to study social sciences at Taipei Imperial University (which was the only institution of advanced studies in Taiwan before 1945, and was re-named National Taiwan University after the war), so most of the wealthiest, best and brightest mostly went to its medical school. Moreover, medical doctors not only practised medicine but some also became leaders of social movements during the Japanese colonial period. As a result, medical practitioners nowadays often have greater influences on Taiwanese people.

Medical doctors' long-standing high socio-economic status and the respect they receive are certainly reasons for some high school students in modern-day Taiwan to study hard and try to get into medical school. Therefore, medical students are usually high performers in many subjects, including mathematics. However, as far as the authors of this paper can observe, mathematics education in medical schools in Taiwan does not make use of their advantages well. Medical students usually have to study only very basic calculus and some bio-statistics, and that is all they learn about mathematics in their training. The philosophy behind this is that, unlike science or engineering students, medical students will not need so much mathematics when they practise medicine in the future. The message sent to medical students, and in turn the message those future doctors may send to the general public in Taiwan, is that mathematics is only there to solve problems, and nothing more.

Problem-solving is certainly one very important reason for anyone to study mathematics, but as most mathematics educators would agree, the discipline may also improve a person's abilities related to conjecture, induction, deduction, abstraction, specialisation and generalisation. Moreover, mathematics is a cultural phenomenon related to histories and philosophies of different civilisations across the face of the earth. Mathematics is not only a science, but also belongs to the humanities. If the personal observation of one of the authors of this paper bears any significance, it would seem that medial university students are more than capable of appreciating the humanistic value of, for instance, music and philosophy, but for science and mathematics, applications and problem-solving are the only value for them. Therefore, if some course in a medical school can change students' beliefs about mathematics, then in the long run medical practitioners may influence the people of Taiwan in regard to the other values of mathematics. Also, many medical doctors in Taiwan's modern history have pursued a political career and become high-profile politicians, so changing the mathematics-related beliefs of medical students might also change the beliefs of some future policy makers. Indeed, what constitutes "good science" or "good mathematics" has often been influenced by politics, as evidenced by many cases in history, such as the competition among several cosmic models during the time of the scientific revolution in Europe (Kuhn, 2003), or the debate between logical and intuitive approaches of Euclidean geometry during Emperor Kangxi's reign in China (Jami, 2012).

Besides, before the beginning of this study, there have been very few studies that discuss mathematical culture courses for non-mathematics majors at the university level. Under these rationales, the main aim of this study was to explore how medical university students' mathematics beliefs changed after taking a liberal arts course in mathematics with an emphasis on the cultural and historical dimensions of mathematics. The authors especially wanted to explore how this course changed medical university students' beliefs about the nature and values of mathematics. In the following, we shall discuss related literature, and then describe the research setting and results we have found.

II. Literature Review

1. The nature and values of mathematics

As mentioned in the previous section, the authors wanted to explore students' beliefs about the nature and values of mathematics. Although it is clear that students might have different beliefs about the values of mathematics, one thing that may need to be clarified is what we mean by the "nature" of mathematics in this paper. When one reads that the authors intend to discuss students' beliefs about the "nature" of mathematics, she might assume that the authors take a Platonic stance to mathematics. Indeed, Plato's realism claims that geometrical objects exist in the "world of being," and the propositions of geometry are objectively true or false, independent of the mind and language of mathematicians (Shapiro, 2000, p. 53). Thus, in Plato's universe mathematical objects do exist and certainly have a nature. However, one can still discuss the nature of mathematics if she takes the stance of, for instance, formalism. One formalist view of mathematics likens the practice of mathematics to a game played with linguistic characters. A radical version of this view asserts outright that the symbols are meaningless (Shapiro, 2000, p. 144). In this view, the nature of mathematics could be seen as mere verbal games. Also, intuitionists' accounts of the nature of mathematics are that it is idealised mental construction (Shapiro, 2000, p. 178). One can even take social constructivists at their word and believe that mathematics is somehow socially constructed (Ernest, 1991, p. 85). Imre Lakatos preached that mathematics, like natural sciences, was fallible, not indubitable; it too grew by the criticism and correction of theories which were never entirely free of ambiguity or the possibility of error (Davis & Hersh, 1987, p. 347). In this view, the nature of mathematics can be seen as a kind of consensus formed after dialogues among individuals and in society. As the reader can see, non-Platonic views of mathematics also discuss the ontology, or nature, of mathematics.

That being clarified, this study did not intend specifically to categorise the medical university students' philosophical stances towards the ontology of mathematics. We simply wanted to see how they thought about "what mathematics is" and how much they agreed with items that described the nature and values of mathematics.

The second dimension of beliefs the authors wanted to explore, as already raised in the last paragraph, is about the "values" of mathematics. This is a dimension that is not completely identical to the "nature" of mathematics. If the nature of mathematics can be briefly described as "what mathematics is," then a simple way of describing the values of mathematics is "what mathematics can do for us." Those who hold an instrumentalist view of mathematics might argue that mathematics is an accumulation of facts, rules, and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts (Ernest, 1994). In this view, the nature of mathematics is necessarily dependent on its applications, and hence the values of mathematics. However, as the reader can easily see that, in many cases, whether one believes that a mathematical entity (such as a right-angled triangle) exists in the Platonic realm of ideals, in human intuition, or in the collection of social constructions, it can pretty much have the same applications (such as measurement by trigonometry). Therefore, the authors of this paper maintained a more cautious stance and treated the nature and values of mathematics as different, though not entirely unrelated, dimensions of mathematics-related beliefs when we did our research and designed the questionnaire (see below in Research Setting).

2. Mathematics beliefs and learning

The authors are aware that, in the field of mathematics education, there are many variations on the concept of belief and belief system, some of whose definitions do not make a clear distinction between belief and those of conception and knowledge (Furinghetti & Pehkonen, 2002). In this paper, we have decided to follow a broad and intuitive working definition of students' mathematics-related beliefs raised in (Op ‘t Eynde, De Corte, & Verschaffel, 2002):

> Students’ mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true, that influence their mathematical learning and problem solving. (p. 16)

Therefore, according to this broad definition, what we mean in this paper by students’ mathematics-related beliefs is their subjective conception of mathematics that may or may not coincide with objective knowledge. They can include but are not limited to students’ prejudice and misconceptions. Students’ mathematics-related beliefs can also include their simple opinions, results of reflections, impressions, deep or superficial feelings, and knowledge without demonstration.

Many studies suggest that students’ beliefs about the nature of mathematics can influence their mathematical learning. According to Schoenfeld’s observation of high school and university students, one’s mathematical worldview shapes the way one does mathematics, and belief systems are one’s mathematical worldview (Schoenfeld, 1985). He also holds that mistaken beliefs about mathematics not only cause students’ conflicting epistemological beliefs about mathematics, but also have negative influences on students’ academic achievements. (Schoenfeld, 1992; 1994). Therefore, he strongly suggests that teachers and educators help students develop appropriate mathematics beliefs. Schoenfeld’s findings resonate with many other studies. Stage and Kloosterman (1991) find that university students with lower achievements in mathematics usually have poorer understanding about the nature of mathematics. Cifarelli and Goodson-Espy (2001) point out that the university students’ epistemological beliefs about mathematics do affect their mathematical learning.

In their paper, Op ‘t Eynde et al. (2002), after reviewing several important twentieth-century publications about mathematics beliefs, also propose a framework for students’ mathematics-related beliefs. In this framework mathematics-related beliefs are described in students’ class context, and are divided into three categories: beliefs about mathematics education (mathematics as a subject, learning and problem-solving, and teaching in general), beliefs about self (self-efficacy, control, task value, and goal-orientation), and beliefs about the social context (the role and functioning of the teacher and the student, and the social-mathematical norms in the classroom, e.g., what counts as a good solution?).

In Taiwan’s local studies, some researchers have also shifted their interests to the field of beliefs. Tang (2009) discusses, among other topics, the beliefs of mathematics intern teachers and their relations to their teaching performances. Wang and Chin (2007) investigate the ways mentors intervene in the mathematics teaching of practice teachers, and the principles and underlying beliefs for their interventions.

Some studies have been done on the social or psychological factors of mathematics teaching and learning in primary and secondary schools in Taiwan, but usually the focuses are on attitudes and anxieties about mathematics. For instance, Wu and Ge (2006) find that students who have success attribution, lower mathematics anxiety, or positive mathematics attitudes, would have higher mathematics achievement, while students who have failure attribution, higher mathematics anxiety, and passive mathematics attitudes, would have lower mathematics achievement. For secondary school students, some studies do address directly to their mathematics beliefs. For vocational high school students, they often believe that mathematics merely consists of numbers, symbols, calculations, and formulae. In other words, it is a group of facts, procedures, and tools. They also believe that learning mathematics requires talent, and that mathematics can be used in real life (Ke, 2013). Another study shows that high school students in Taiwan who have decided to study science, engineering, or medicine believe that mental exercises and professional necessities are the most important values of mathematics, but still they do not really see how high school mathematics can be applied in real life (Chou, 1999). This study also tells us that, even at the end of the 20th century, when senior high school and university education in Taiwan were more or less focused not on the general public but on the elites, students believed that the main values of mathematics were about application, and its humanistic values were not considered. Some of those students who aimed to study science or medicine would actually become medical practitioners, so it is reasonable to believe that they still hold the same beliefs, if they have had no chances to be exposed to the humanistic side of mathematics in their tertiary-level education.

Although there are already some studies about beliefs in primary and secondary mathematics education, very few studies focus on Taiwanese tertiary students' mathematics beliefs. One important work, (Liu, 2007), shall be discussed in the next sub-section. In another study, Hong (2009) explores gender issues among students in universities of technology. In her work, she concludes that the stereotype that mathematics teaching environments unsuitable for female students would lower their willingness to pursue mathematics-related professional development. This is also a case in which culture affects mathematics beliefs and behaviour.

From the aforementioned literature review, we can see that belief was a hidden variable in mathematics education throughout the previous decade (Leder, Pehkonen, & Törner, 2002), but since then researchers have paid more and more attentions to the research of mathematics beliefs in different levels of mathematics teaching and learning. This paper also aims to enrich research in this area, through investigating the effects of a course in mathematics with an emphasis on the cultural and historical dimensions.

3. Mathematics learning and history of mathematics as a form of mathematical culture

The title of this paper contains the term “mathematical culture”, by which the authors broadly mean any aspect of human creation that is related to the academic field we now refer to as “mathematics,” such as logical thinking, or the applications of mathematical methods in architecture, art, astronomy, or business. The reason the authors hold this broad view is that there has always been a “humanist” trend in mathematics, in which philosophers, such as Aristotle, John Locke, David Hume, John Stuart Mill, and many others, saw mathematics as a human creation (Hersh, 1997, p. 183). These philosophers applied mathematics to many aspects of human society, such as science, art, and critical thinking. Practical, scientific, aesthetic, and philosophical interests have all shaped mathematics, which in turn has moulded modern culture (Kline, 1953). There is an abundance of instances of mathematical culture in human history, including how philosophers discussed Nature, and how ancient people solved real-world problems with mathematics. It is widely accepted that modern mathematics has a “cultural basis” (e.g., Wilder, 1950), and scholars believe that history can be used as a “crossroad” of mathematical culture and mathematics teaching (Furinghetti & Paola, 2003). Therefore, the authors of this paper also wanted to use the history of mathematics to design a mathematical culture course.

What can the history of mathematics do for mathematics teaching and learning? There are many answers. Furinghetti and Paola (2003) believe it can stimulate students’ interests in mathematics. Tzanakis and Arcavi (2000) propose that using history in mathematics teaching helps learning, and it gives teachers a chance to have an alternative view on the nature and development of mathematical activities. Teachers may learn to appreciate mathematics as a cultural phenomenon. Through different solutions to the same problem in different times and places, students can also appreciate the evolution of mathematical knowledge (Horng, 2000).

In Taiwan’s local research and education practices, the integration of certain historical topics of into mathematics teaching, learning, and teacher education has been advocated since the 1980s (e.g., Horng, 1984), and many primary and secondary school teachers have been trying to use history to assist their teaching, many using worksheets and pre-modern texts as tools (Su, 2007; Tsai & Su, 2009). Animation, storytelling, and theatrical performances are more recent practices mainly used in primary schools (Su, 2011). For secondary school mathematics, under the guidance of the research of HPM (relations between History and Pedagogy of Mathematics), it is possible for the teacher or students to have a certain level of liberation in the knowledge content of the textbook, to untie or overthrow the more conventional aspects of the textbook (Horng & Su, 2007). There is also evidence that reading pre-modern mathematical texts and reflections improves in-service teachers' pedagogical content knowledge (Su & Ying, 2014).

Meanwhile, Liu (2003) proposes several reasons to use the history of mathematics in school curricula. They include: (1) history can help to increase motivation and to develop a positive attitude towards learning; (2) historical problems can help develop students; mathematical thinking; (3) history reveals the humanistic facets of mathematical knowledge; (4) history gives teachers a guide for teaching.

Most of the studies tell us that history of mathematics can show students how mathematical knowledge developed and evolved, and help them appreciate mathematics as cultural activities created by different peoples. However, these studies into the influence of history on students' learning and beliefs, whether local or international, mainly focus on school mathematics. Very few studies explore university teaching and learning. Liu (2007) is one of very few papers that addresses directly the issue of the influence of history on university students' beliefs about mathematics. This study describes a history-oriented calculus course and the development of the views of mathematics held by students' in that course. Its results indicate that most of the students considered mathematics thinking to be the processes involved in calculation and formulae at the very beginning, nonetheless expressing different viewpoints after finishing the course. They attempted to connect the context of mathematicians' problem solving with the characteristics of mathematics thinking. Although students strongly held a consistent instrumentalist view of mathematics, at the end of the course they were more likely to recognise mathematics as the continuity and inheritance of knowledge and the role of mathematicians in its development. Furthermore, this study also emphasises the importance of students' understanding of the nature of mathematics. Kjeldsen (2011) shows that using history as a means to teach university students differential equations helps them understand the special nature of mathematical thinking, and the idea that mathematics and physics closely interact with each other. However, among the few studies about the relations between history and university students' beliefs about mathematics, none of them studies students in medical universities. Two of the abovementioned studies, (Liu, 2007) and

(Hong, 2009), address cases of universities of technology in Taiwan, but other than that, it would seem that the mathematics belief system held by students in non-comprehensive universities in Taiwan is still an open problem. Knowing the possible advantages of the history of mathematics and the current status of research, with the rationale introduced in the beginning of this paper, it should be easy for the reader to see why the authors of this paper wanted to devise a university mathematics course with an emphasis on the cultural and historical dimensions of mathematics, and to investigate whether it could change the stereotypical beliefs about mathematics that might be held by students in medical universities.

III. Research Setting

This study used a single-group pretest-posttest design. The research tools of this study included a liberal-arts mathematics course with an emphasis on the history and culture of the discipline, and a questionnaire used in the pre-test and the post-test. The same group of students took the pre-test, participated in the teaching experiment, and finally took the post-test. A total of 131 students took the course, and they were separated into two classes. One of the authors was the lecturer in the teaching experiment of both classes. In what follows we shall describe the research setting in detail.

1. Subjects

The subjects of this study were students at a medical university in Taiwan. These students took an elective liberal-arts course about mathematics in the second semester of Taiwan’s 2012-13school year (February to June 2013) that was designed to be the teaching experiment in this study. Students in this course consisted of students in all twelve undergraduate schools and departments of this medical university. Their majors were medicine, dentistry, pharmacy, nursing, public health, and other related disciplines (the professional training for medicine and dentistry are undergraduate programs in Taiwan, but it takes longer time for students to earn a degree; a degree in medicine usually takes six to seven years to complete, while that in dentistry usually takes six years). Most of the 131 students in the course were in their first or second year in university, although a few students were in their third or higher years.

2. Research tools

(1) Mathematics beliefs of the authors and the lecturer

As one of the authors was also the lecturer of the teaching experiment, the mathematics beliefs of the authors, and especially those of the lecturer, might or might not have influenced the results of the experiment. Therefore, it may be necessary to describe the beliefs held by the authors of this paper. As mentioned earlier, this study was not specifically intended to categorise the students' particular philosophical stances, and the authors tried not to hold a clear assumption about the ontology of mathematics. Interestingly, on the one hand, many of the greatest mathematicians in history have firmly taken a Platonic stance, most notably from Euclid in the Hellenistic period to Kurt Gödel in the modern era (Lloyd, 1975; Parsons, 1995), while, on the other hand, some historians, sociologists, and educators of mathematics tend to emphasise on the social and cultural dependence of mathematics (e.g., Ernest, 1991; Jami & Han, 2003). So arguing for or against any particular stance would be just as easy (or as difficult) as any other. In fact, some scholars even argue that a working mathematician might hold seemingly contradicting beliefs about the nature of mathematics for different purposes (Davis & Hersh, 1987, p. 321). For the authors of this paper, our training and backgrounds are closer to those of historians and educators than pure mathematicians, so we lean more to the side of intuitionism and social constructivism on the spectrum of the ontology of mathematics, although the lecturer did not specifically argue for or against any ontological stance in class.

(2) The teaching experiment

The course used as the teaching experiment, titled "Mathematical Thinking in the Multicultural Contexts", was taught during a seventeen-week semester, with two hours of class time each week. This course tried to introduce to students the mathematical activities in several pre-modern and early-modern civilisations. Students were exposed to mathematical topics that they had learned before but presented in historical context, some topics of advanced mathematics they might not have seen in high school, and some applications of mathematics in different societies. There were also examples to distinct solutions to the same problems by mathematicians in different civilisations, such as comparing how Liu Hui (3rd-century China) and Euclid's *Elements* discussed the volume of a pyramid. There was also a mid-term project that had the students create an art work with some mathematical concepts in it. The reader may refer to the appendix for the syllabus of this course.

In what follows, we shall use weeks 6 to 8, 11, 14, 15, and 17 to show how this course was taught and how students learned in it.

In weeks 6 and 7, the main theme was to show that in different ancient civilisations, they tackled similar problems with distinct approaches. The protagonist in East Asia was Liu Hui in the 3rd century, and his works were compared with those of Euclid and Archimedes. The mathematical contents involved were the arguments for the area of the circle and the volume of the pyramid. For both cases, Liu Hui used methods of limits and infinitesimals to argue for his formulae in his commentary on the *Nine Chapters* (the *Nine Chapters of Mathematical Art*, or *Jiuzhang suanshu*, was considered one of the most important mathematical canons and the paradigm for the learned practice of mathematics in pre-modern China; the reader may refer to, for instance, [Chemla & Guo (2004)] for a detailed translation and analysis of the text). In the case of the circle, Liu Hui argued that if one "cut" the circle—that is, constructed inscribed regular polygons—in certain ways, the areas of the inscribed polygons would always equal "half-circumference multiplied by half-diameter." And if one continued to cut until one could not cut any more, then the inscribed polygons would coincide with the circle, and the area formula of the circle was just the same as those of the inscribed polygons. Similarly for the pyramid, Liu Hui argued that if one dissected the pyramid in certain ways, rearranged some solids and dissected the rest, and then continued the process in an infinite manner, one would reach the conclusion that the volume of a pyramid was one third of the cuboid with the same length, width and height. However, for both cases, the Greek mathematicians used reductio ad absurdum to prove the same results. The reason they did not use methods of limits and infinitesimals is very likely due to the fact that Greek philosophers avoided talking about "infinity" for it might lead to logical paradoxes. The point in weeks 6 and 7 was to show the students that due to different cultural backgrounds, mathematicians used distinct approaches for the same problems. Ancient East Asian scholars used more intuitive approaches, while the Greeks, for logical consistencies, avoided arguments about infinity and used finite steps with reduction to absurdity to reach the same result.

In week 8, the point was also about contrast. The diagrams in Greek mathematics, due to the influence of Platonic philosophy, are always "static." The focus of the diagrams in propositions in Euclid's *Elements* is on their structures. A mathematician would refer to different components of the diagrams to prove the wanted results. The focus of the diagrams in East Asian texts is not the diagrams themselves, but the transformations of the diagrams. For instance, one would transform an original diagram of a triangle into that of a rectangle to show the area formula of a triangle. As the reader can see, in weeks 6 to 8, students were exposed to different patterns of mathematical thinking. They could, in the process, learn to appreciate different approaches of thinking.

In week 11, students were asked to present their artistic creations related to mathematics. At first glance, one might wonder why a task about the arts was put into this course of mathematical culture. The reason was because the arts—we use the term in a broad sense that includes visual arts, performance arts, literature, music, etc—have always been one of the many domains of life that implicitly make use of mathematics, and by creating artistic works, students might feel how mathematics were applied in different aspects of human cultures. In the reality of the course, the authors must admit that not all of the students made very interesting creations. However, some students successfully brought enough mathematical and artistic elements into their works. For instance, some students wrote mathematical novels. Some created suspense stories in which the readers/viewers had to use their mathematical knowledge to crack puzzles. Some invented board games that required mathematics to win. Through this process, they could feel that mathematics could be linked to artistic creations. The grading of this task could also link students' aesthetic experience to mathematics. The grade for each artistic creation was given by both the lecturer and students. When students graded others' works, they had to consciously look for mathematical elements in the creation, and consider if they fit their own standard of "beauty." In this way, students learned to appreciate art through glasses of mathematics.

In weeks 14 and 15, the lecturer talked about calendrical astronomy and how several of our calendars were created in different cultural backgrounds. Behind the fascinating stories was the intent of the lecturer to show students that mathematics could and actually did solve problems about the real world, and the results—the calendars—were deeply embedded in our cultures.

In week 17, the final written examination was mainly about the historical and mathematical knowledge they learned in class. For each question, students had to give their answers and explain how they reached this answer. That means they often had to give background information in addition to mathematical arguments. Students were allowed to bring hardcopy or electronic references to the examination. So they did not need to memorise most of the factual knowledge, but they would have to understand the logic and arguments used by pre-modern mathematicians to get good grades in the examination. Beside this kind of questions that tested students' knowledge about the history of mathematics, there was also the final question, called the "reflexive question" by the authors, which asked if students could give an example of the same problem being tackled in distinct manners by both European and East Asian mathematicians, and if they could use this as an example to show the differences between the two civilisations. This was not an easy question, but some students managed to present good examples and arguments. And although, strictly speaking, this question could not be regarded as a qualitative question directly about mathematics-related beliefs, the students' answers could still serve as circumstantial evidence when the authors analysed the results of the quantitative questionnaire introduced in the next sub-section.

(3) The questionnaire

The questionnaire used in the pre-test and post-test consisted of 20 Likert-scale questions, each of which was a declarative sentence about mathematics. The responses from which students could choose for each statement formed a set of typical seven-level Likert items: "strongly agree," "agree," "slightly agree," "neither agree nor disagree," "slightly disagree," "disagree," and "strongly disagree." For each question, the student had to choose one and only one item from the seven. Students' responses were then transformed into scores for analysis, with the highest score of 7 for "strongly agree," 6 for "agree," 5 for "slightly agree," 4 for "neither agree nor disagree," 3 for "slightly disagree," 2 for "disagree," and the lowest score of 1 for "strongly disagree." The questions in the pre-test and the post-test were identical, so we could see the changes in students' beliefs.

As mentioned before, Op 't Eynde et al. (2002) discuss a useful framework for mathematics-related beliefs. However, since it mainly deals with primary and secondary school students, we believe it was necessary for us to modify it according to our purposes. In this tertiary-level course for our teaching experiment, the intellectual history played as important a role as the social history and culture of mathematics, so we believed that the structure for our questionnaire must address the (intellectual) nature of mathematics at least as much as the (social and personal) values of mathematics. Therefore, before the teaching experiment began, the authors divided the questionnaire into two dimensions (nature and values), and devised questions accordingly.

During the developmental period, the questionnaire was examined by three external experts whose respective expertise was in beliefs and affects, curriculum, and HPM in order to ensure the validity of the questions in the questionnaire. One of the external experts, whose specialty was in curriculum, was invited to join the authors' discussions and offer comments when we were developing the course and the questionnaire. After the first version of the questionnaire was finished, it was sent to the other two experts for review. We told them about the items of the questionnaire and what we really wanted to inquire about students' beliefs, and they gave us comments about how we could change our wording so it would be easier for students to understand. We later changed many words usages and expressions, so that what students would read was closer to what the authors would like to inquire about and to the contents of the course. For instance, we wanted to see if students believed that exemplification justified a mathematical proposition, and the original statement was "By checking a large number of examples one can justify mathematical knowledge." One of the external experts told us that this sentence was vague and suggested changes. Eventually the statement was modified to become "If an assumption in mathematics fits most situations, then it is justified".

Finally, the design of the questionnaire was finished. Table 1 shows the structure of our questionnaire, in which questions were separated into two dimensions of beliefs—the "nature of mathematics" and the "values of mathematics"—each of which contained 10 questions. The statement mentioned at the end of the previous paragraph is question 17 in Table 1.

Table 1

The Structure of the Questionnaire

Dimensions	Questions
Nature of mathematics	1. Mathematics is a discipline described with symbols. 3. Mathematics is a discipline composed of procedures and formulae. 5. Mathematics is a discipline that finds general principles from individual facts. 7. The general principles found in mathematical research can be applied to all kinds of situations that fit the conditions of the principles. 9. Mathematics is a discipline that constructs common models from complex phenomena in reality. 11. Results calculated with mathematics can accurately explain many phenomena. 13. Mathematics is a rigorous and logical discipline. 15. Proof is the only method that justifies mathematical knowledge. 17. If an assumption in mathematics fits most situations, then it is justified. 19. Mathematics is objective truth.

(continued)

Table 1 (continued)

Dimensions	Questions
Values of mathematics	2. The research results of mathematics can help human beings describe phenomena of the real world.
	4. The research results of mathematics can be used as tools to solve problems in other fields of study.
	6. Learning mathematics can help us solve problems encountered in daily lives.
	8. Learning mathematics can cultivate our logic and reasoning.
	10. Learning mathematics can cultivate our creativity.
	12. Learning mathematics can cultivate our observing ability.
	14. Learning mathematics can increase our knowledge.
	16. Learning mathematics can elevate our ability to judge things around us.
	18. Learning mathematics can improve our sensibility to beauty.
	20. Learning mathematics is helpful to our career development.

Since questions in the same dimensions have similar wordings, we gave them an iterated order—odd-numbered questions for the nature of mathematics and even-numbered ones for the values of mathematics—to reduce the disturbance that the wordings might have caused. In the next section, we will discuss the findings of our study seen in the questionnaire.

IV. Results

After deleting the samples with missing values and invalid questionnaires, we collected 112 valid questionnaires in the pre-test and 121 questionnaires in the post-test. However, since this was a elective course, there were some changes in participating students between the beginning and the end of the course. Since we needed to compare students' scores between pre-test and post-test, we excluded the data of those students who only took one of the two tests. Eventually, there were exactly 100 students who participated in both the pre-test and post-test. The average score of each question in each test was computed and recorded. We used Paired-Samples *T* Test (α = 0.05) to compare the averages of two paired samples that contain these 100 students. We found several interesting results, some of which are discussed below. The reader may refer to Table 2 in the following discussions (the *p*-values less than .05 are marked with a "*").

Table 2

Average Scores and Standard Deviations of Questions in the Pre-test and the Post-test (N = 100)

Question number	Pre-test *M*	Pre-test *SD*	Post-test *M*	Post-test *SD*	*t*	*p*-value	*95%* CI *LL*	*95%* CI *UL*	effect size
1	5.48	1.01	5.39	1.24	0.736	.464	-0.15	0.34	0.15
2	5.39	1.10	5.90	0.79	-4.164***	.000	-0.75	-0.26	-0.83
3	5.18	1.23	5.21	1.21	-0.245	.807	-0.28	0.22	-0.05
4	6.40	0.68	6.46	0.56	-0.743	.459	-0.20	0.09	-0.15
5	5.52	0.90	5.66	0.93	-1.094	.277	-0.40	0.12	-0.22
6	5.37	1.21	5.58	1.00	-1.760	.081	-0.44	0.03	-0.35
7	5.09	1.30	5.37	1.08	-2.031*	.045	-0.56	-0.01	-0.41
8	6.28	0.77	6.32	0.89	-0.335	.739	-0.24	0.17	-0.07
9	5.33	1.08	5.48	0.93	-1.224	.224	-0.40	0.09	-0.24
10	4.77	1.12	5.21	1.15	-3.560**	.001	-0.69	-0.20	-0.71
11	4.86	1.21	5.12	1.08	-2.077*	.040	-0.51	-0.01	-0.42
12	5.43	1.13	5.65	1.05	-1.762	.081	-0.47	0.03	-0.35
13	5.87	0.93	5.95	0.87	-0.942	.348	-0.25	0.09	-0.19
14	5.55	0.92	5.64	0.96	-0.886	.378	-0.29	0.11	-0.18
15	3.95	1.57	4.31	1.30	-2.347*	.021	-0.67	-0.06	-0.47
16	5.20	1.11	5.40	1.03	-1.694	.093	-0.44	0.03	-0.34
17	4.19	1.24	4.58	1.25	-2.473*	.015	-0.70	-0.08	-0.49
18	4.37	1.31	4.91	1.25	-3.704***	.000	-0.82	-0.25	-0.74
19	5.19	1.28	5.24	1.24	-0.378	.706	-0.32	0.21	-0.08
20	4.83	1.19	5.02	1.20	-1.527	.130	-0.44	0.06	-0.31

Notes: *$p < .05$. **$p < .01$ ***$p < .001$

Further analysis showed that after taking this course, students agreed more on the following: mathematics can help human beings describe phenomena in the real world (q.2, p = .000); general principles found in mathematical research can be applied to all kinds of situations that fit their conditions (q.7, p = .045); learning mathematics can cultivate our creativity (q.10, p = .001); results calculated with mathematics can accurately explain many phenomena. (q.11, p = .040); proof is the only method that justifies mathematical knowledge (q.15, p = .021); if an assumption in mathematics fits most situations, then it is justified (q.17, p = .015); learning mathematics can improve our sensibility to beauty (q.18, p = .000). The seven questions listed in this paragraph, although show significant changes, do not have large enough effect sizes. This could be a result of the relatively small number of samples (100 students), which is a limitation of this study. This being said, we can still see that an important value of this teaching experiment was the change it may have caused in students' beliefs. In what follows, we shall discuss the reasons for significant and non-significant changes in the two dimensions.

IV. Discussions

1. Changes in beliefs about the nature of mathematics

In the aspect of the nature of mathematics, six questions had no significant changes, while four had significant changes after the teaching experiment. Q.1, 3, and 5 were about contents and representations of mathematics—symbols, procedures, formulae, and general principles. Since, during the course, the lecturer had shown multiple representations of various kinds of contents in mathematics created by different cultures, students' beliefs about contents and representations were neither strengthened nor weakened, and hence there were no significant changes. Q.9 was about modelling. The non-significant change could be due to the fact that the course seldom talked about how ancient cultures created models that represented a part of the real world, and the term "model" was never mentioned in class. Q.13 asked if students believed that mathematics was rigorous and logical. For the reflexive question in the final written examination, many students answered that the Greeks put more emphasis on the logical structure of mathematics, while Chinese argumentations appealed more to intuition. Since there were two major paradigms with different emphases on formal rigour, their beliefs about the logical method of mathematics were again neither strengthened nor weakened. A similar explanation can be argued for the non-significant change in q.19, since very different methodologies and results were created by the two paradigms, students were not more prone to believe that mathematics was objective truth.

For the four questions that had significant changes, we can offer some interesting explanations as well. Results showed that after taking the course, the students were more prone to believe in the "generalisation" thinking of mathematics (q.7), that is, that general principles can be applied to different situations. Students also agreed more on the claim that "results calculated with mathematics can accurately explain many phenomena" (q.11). This could very well be due to their exposure to the topic of calendrical astronomy.

However, in terms of how to justify mathematical knowledge, the scores of both q.15 (proof) and q.17 (exemplification) were increased, showing that this course did not clarify for students the difference between "the context of justification" and "the context of discovery." In the context of discovery, it is reasonable for a person to believe in some assumption if it is verified in most situations she encounters or can think of; however, in the context of justification, the only mathematically acceptable method to justify a proposition is through logic. In the teaching experiment, the reader can see from the appendix and discussions in section III that the focus was mainly on approaches used by different ancient civilisations to tackle similar problems, and also on the link between mathematics and the physical world, but not on how ancient philosophers and modern mathematicians developed logic and ways to justify mathematical knowledge. This is a point that lecturers of similar courses should keep in mind when they teach them: ancient mathematicians used experiments, manipulations of mental or physical objects, or even mere intuition to discover some propositions, but then they had to use logic to justify them.

A final remark about the nature of mathematics is that, although the authors and the lecturer stood closer to intuitionist and social constructivist views on the spectrum of the ontology of mathematics, students' beliefs about mathematics being objective truth and about the logical method of justification were not weakened in any way. This could mean that in this course of mathematical culture, even if the lecturer presented different aspects of mathematics created in different ancient cultures, the lecturer's own philosophical stance would not alter students' ontological beliefs in any drastic way. This could be a topic of future research.

2. Changes in beliefs about the values of mathematics

There were seven questions about which students had no significant changes in their beliefs. The initial observation that one of the authors made, that medical university students saw mathematics as a useful tool, was already confirmed after the pre-test. Most of the average scores of the even-numbered questions were relatively high, and two of them were even more than 6 (q.4 and 8). Q.4 was about mathematics as a tool to be applied in other disciplines, and q.8 was about mathematics used to train one's reasoning. Since their pre-test scores were already high, and in the course the lecturer also gave examples of mathematics applied to many fields of mathematics, such as engineering and astronomy, their beliefs about the applicative values of mathematics maintained at a high level.

As for significant changes, after taking the course, the students agreed more on q.2, 10, and 18. Q.10 and 18 talked about creativity and sensibility to beauty, neither of which had been a value in traditional stereotypes of mathematics. In the course, the lecturer gave examples to show how ancient scholars observed and described Nature (such as the case of calendar-making). The authors believe that this was the reason the students' beliefs changed. Moreover, the lecturer let students compare different thinking on the same problem and he asked the students to create artwork with mathematical concepts. In the reflexive question, many students gave examples of different approaches to tackling similar problems, such as demonstrating the area formula of the circle or the volume formula of the pyramid. These differences strengthened the idea of creativity in mathematics. Besides, their artistic works gave them a different way to use mathematics. These were clearly the reasons for which students realised the values of creativity and sensitivity to beauty.

A final topic that this paper may probably address is whether changes in the beliefs of one dimension in this study might affect the other. Unfortunately the evidence the authors found could not support or refute the claim. As mentioned in the literature review, the authors of this paper treated the nature and values of mathematics as different but related dimensions. Q.9 and 11 in the dimension of the nature of mathematics could be seen as related to the values of mathematics, but students' beliefs about only one of them was significantly changed. Although the authors did not see strong evidence of related changes, this topic can certainly be included in future studies of university students' beliefs about mathematics.

Summarising the whole study, the authors believe that with the evidence, a liberal-arts mathematics course with an emphasis on the culture and history of the discipline can indeed change a part of medical university students' beliefs, especially regarding the link between mathematics and creativity and sensibility to beauty, both of which belong to the humanistic aspect of mathematics. Of course, this preliminary exploratory study still has many limitations, such as some possible threats to its internal validity and the small sample size, so the reader should not extrapolate too much about the influences of the proposed course. However, we did see some changes in students' beliefs and hope in the long run that these students could influence the opinion of the public. Also, further modifications can be made about the context of discovery and that of justification, so the contents of the course can still stay close to the modern practice of mathematics. Finally, all these results can serve as references for university lecturers and mathematics educators for future courses and for research into the mathematics-related beliefs held by students in different non-comprehensive universities and in various professional aims.

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Appendix: Syllabus of the teaching experiment.

Week	Theme
1	Introduction to the course.
2	Pre-test administration, unit fractions, the sexagesimal and other numerals in ancient civilisations.
3	Greek philosophy of mathematics, Euclid's *Elements* and its contents.
4	Euclid's *Elements* and its influences.
5	Archimedes, Apollonius and Heron.
6	Mathematics in early imperial China, The *Nine Chapters of Mathematical Art*, and Liu Hui's commentary, Archimedes vs. Liu Hui.
7	Liu Hui's and the Zu family's methods of limits and infinitesimals, Euclid vs. Liu Hui.
8	Chinese algebra in the 13th century: *Tianyuan shu*, the use of geometrical diagrams in Greek and in China, adversary vs. authority.
9	Mid-term break
10	Partition of inheritance, geometrical solutions of quadratic equations.
11	Students (separated into groups of 7-9 students) present their art creations.
12	Cubic equations, the social status and interactions of Renaissance mathematicians and nobles.
13	Japanese mathematics in the Edo period (17th to 19th centuries), mathematical competitions in the temples and shrines.
14	Basic calendrical astronomy.
15	The making of Julian calendar, Gregorian calendar, and the East-Asian lunisolar calendar.
16	The invention of calculus, the histories of "integration" and "differentiation", the arithmetisation of analysis, the fundamental theorem of calculus.
17	Final written examination.